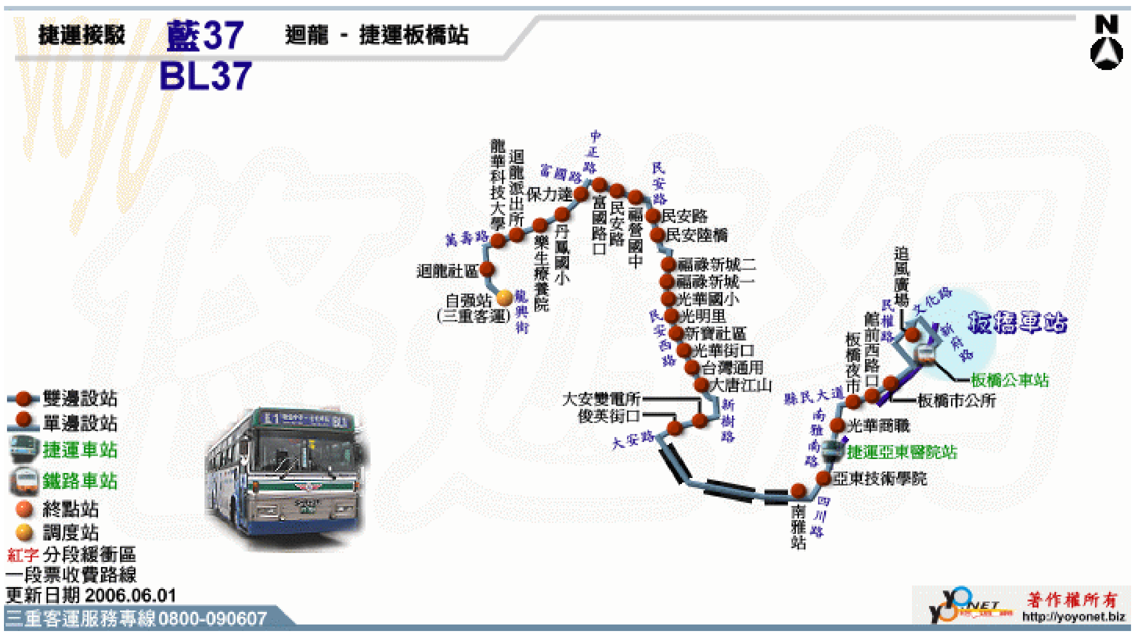
Optimization of a feeder-bus route design by using a multi-objective programming approach



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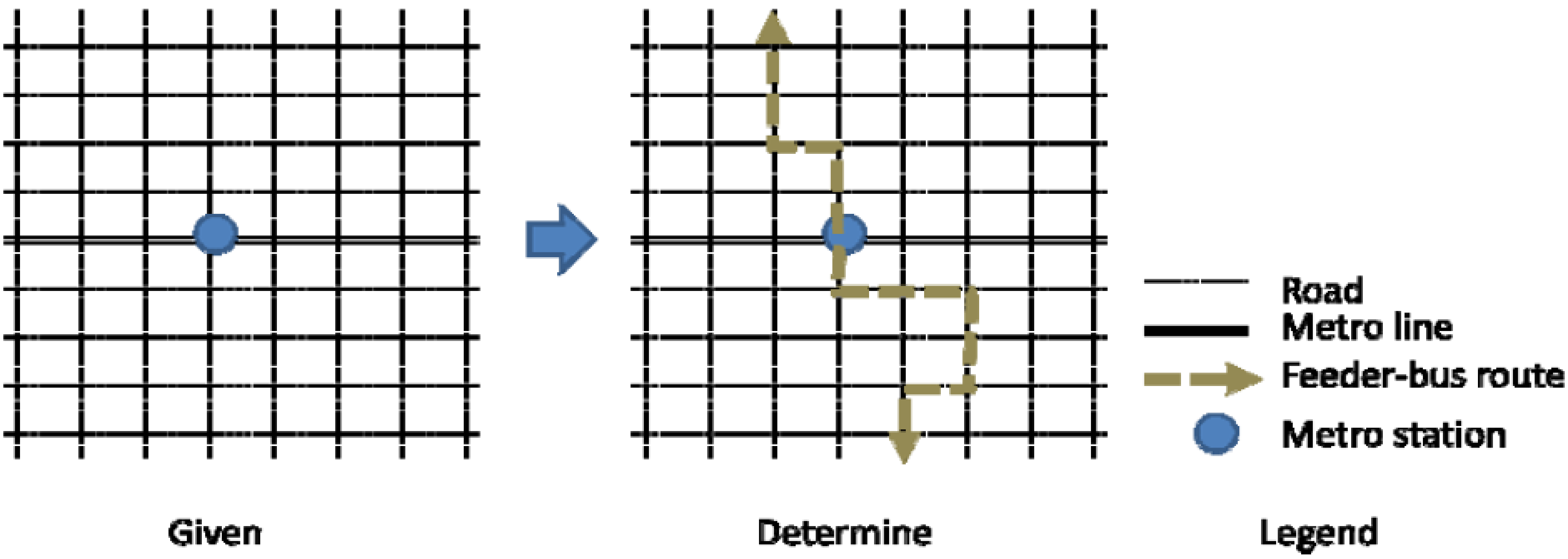
Outline

- Introduction
- Modeling concepts
- Model formulation
- Applications

Feeder-bus services at metro stations



Achievements in literatures

Analytic approaches

- Wirasinghe *et al*. (1977), Wirasinghe (1980), Kuah and Perl (1988), Chien and Schonfeld (1998), Chien and Yang (2000) and Chien *et al*. (2001)

Network approaches

- **FBNDP**
  - Kuah and Perl (1989); extended by Chang and Chang (1994), Martins and Pato (1998), Kuan *et al*. (2004; 2006), Mohaymany and Gholami (2010) and Sun *et al*. (2011)
- **HFRGA**
  - Shrivastava and Dhingra (2001), extended by Shrivastava and O'Mahony (2006; 2007; 2009) and Song and Liu (2011)

Two issues in network approaches

$$\text{Minimize } Z(X, Y, F) = \sum_{j=I+1}^{I+J} C_{js} \sum_{i=1}^{I} Q_i Y_{ij} + 2\lambda_0 \left[\sum_{k=1}^{K} F_k \sum_{i=1}^{I} \sum_{h=1}^{I+J} L_{ih} X_{ihk} \right] + \frac{\lambda_r}{2U} \sum_{k=1}^{K} \left(\sum_{i=1}^{I} \sum_{h=1}^{I+J} L_{ih} X_{ihk} \right) \left(\bar{Q} + \sum_{i=1}^{I} \sum_{h=1}^{I+J} Q_i X_{ihk} \right) + \lambda_w \left[\sum_{k=1}^{K} \frac{1}{2F_k} \sum_{i=1}^{I} \sum_{h=1}^{I+J} Q_i X_{ihk} \right]$$

subject to:

(i) $$\sum_{k=1}^{K} \sum_{h=1}^{I+J} X_{ihk} = 1; \quad i = 1, \dots, I$$

(ii) $$\sum_{i=1}^{I} \sum_{j=I+1}^{I+J} X_{ijk} \leqslant 1; \quad k = 1, \dots, K$$

(iii) $$\sum_{h=1}^{I+J} X_{ihk} - \sum_{m=1}^{I} X_{mik} \geqslant 0; \quad i = 1, \dots, I; \; k = 1, \dots, K$$

(iv) $$\sum_{i \notin H} \sum_{h \in H} \sum_{k=1}^{K} X_{ihk} \geqslant 1; \quad \forall H$$

(v) $$\sum_{h=1}^{I+J} X_{ihk} + \sum_{i=1}^{I} X_{ijk} - Y_{ij} \leqslant 1; \quad i = 1, \dots, I; \; j = I+1, \dots, I+J; \; k = 1, \dots, K$$

- **Single objective: Concerns of stakeholders were incompletely addressed**

(vi) $$\sum_{i=1}^{I} Q_i \sum_{h=1}^{I+J} X_{ihk} \leqslant cF_k; \quad k = 1, \dots, K$$

(vii) $$\frac{2c}{\rho U} \sum_{k=1}^{K} F_k \sum_{i=1}^{I} \sum_{h=1}^{I+J} L_{ih} X_{ihk} \leqslant \text{ASH}$$

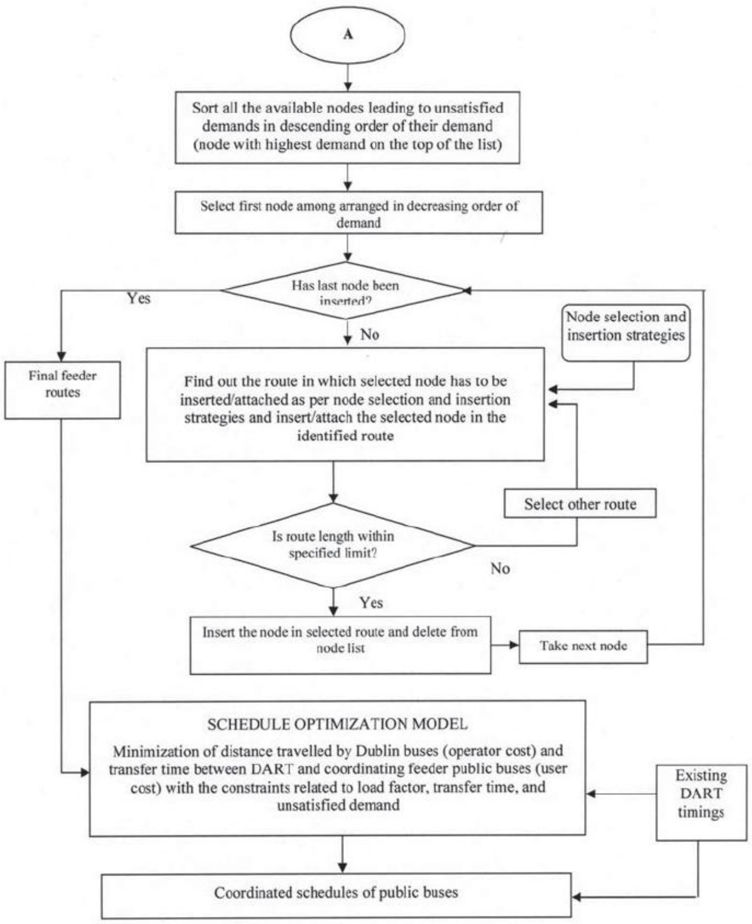
(viii) $$\sum_{i=1}^{I} \sum_{h=1}^{I+J} L_{ih} X_{ihk} \leqslant D_k; \quad k = 1, \dots, K$$

(ix) $$X_{ihk} = 0, 1; \quad i = 1, \dots, I; \; j = I+1, \dots, I+J; \; k = 1, \dots, K$$

$$Y_{ij} = 0, 1; \quad i = 1, \dots, I; \; j = I+1, \dots, I+J$$

$$F_k \geqslant 0; \quad k = 1, \dots, K$$

Two issues in network approaches



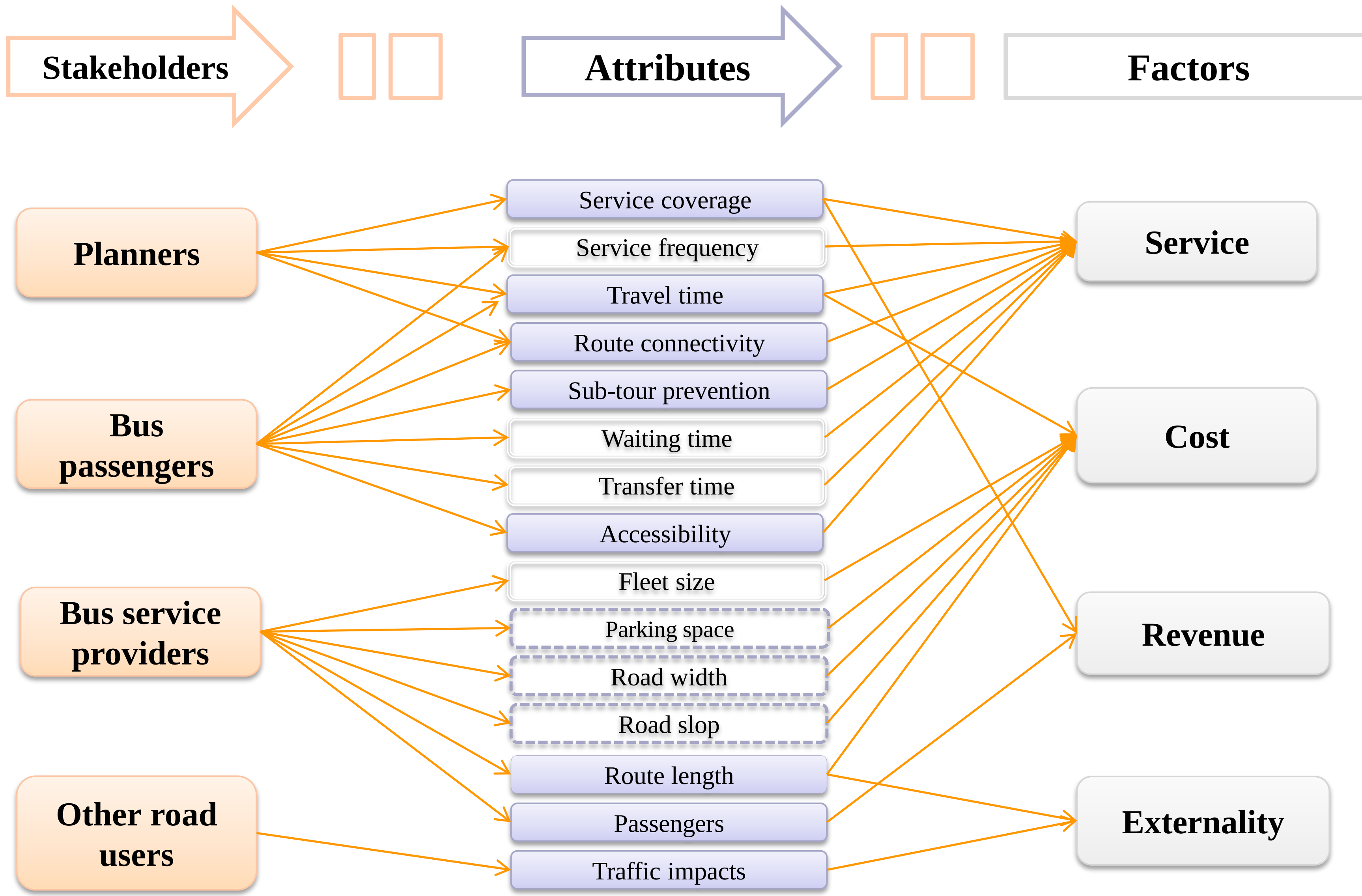
- Developing better algorithms rather than improving models

Purposes

- Developing a feeder-bus route design model for a metro station in developed urban areas
- Applying the model to the metro station G9 in Taichung, Taiwan

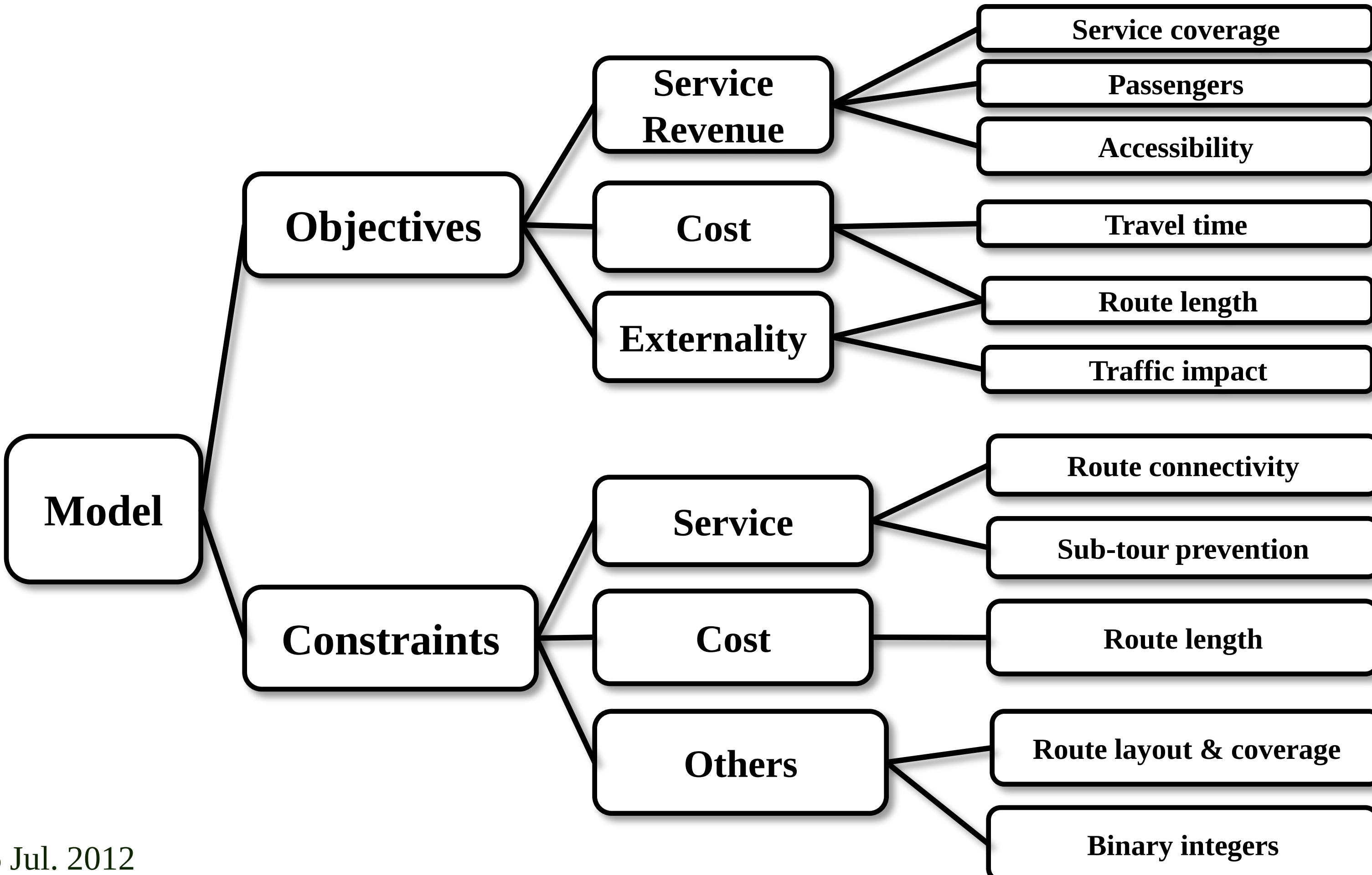
Stakeholders & concerns

- Literature review
- Field interviews
  - Bus route planners in local governments
  - Manager of a major bus company
  - Feeder-bus riders
  - Car-driver



Pre-modeling | In-modeling | Post-modeling

Model framework



Objectives

- **Maximizing service coverage**

$$\text{Max } f_1 = \sum_k (a_k Y_k)$$

s.t.

$$Y_k \le \sum_h \sum_{ij \in S_k} X_{ijh}\,,\ \forall\, k$$

- **Minimizing the maximum route travel time of all routes**

$$\text{Min } f_2 = \alpha$$

s.t.

$$\alpha \ge \sum_{ij \in Z_h} \left(u_{ij} X_{ijh}\right), \forall\, h \in H$$

- **Minimizing the total length of planned routes**

$$\text{Min } f_3 = \sum_h \sum_{ij \in Z_h} \left(d_{ij} X_{ijh}\right)$$

Constraints

■Connectivity along a route

$$\sum_{i\in N_j} X_{ijh} - \sum_{l\in N_j} X_{jlh} = \left\{ \begin{array}{l} -1,\ \ \text{if } j = s_h \\ \ \ 0, \forall j \in Z_h, j \neq s_h, t_h \\ \ \ 1, \text{if } j = t_h \end{array} \right\}, \forall h \in H$$

■No sub-tour

$$\sum_{i\in N_j} X_{ijh} \leq 1,\ \forall\, j \neq s_h, t_h;\ \forall\, h \in H$$

$$\sum_{l\in N_j} X_{jlh} \leq 1,\ \forall\, j \neq s_h, t_h;\ \forall\, h \in H$$

Constraints

- **Length upper bound of a route**

$$\sum_{ij \in Z_h} (d_{ij} X_{ijh}) \leq D_{\max}, \forall\, h \in H$$

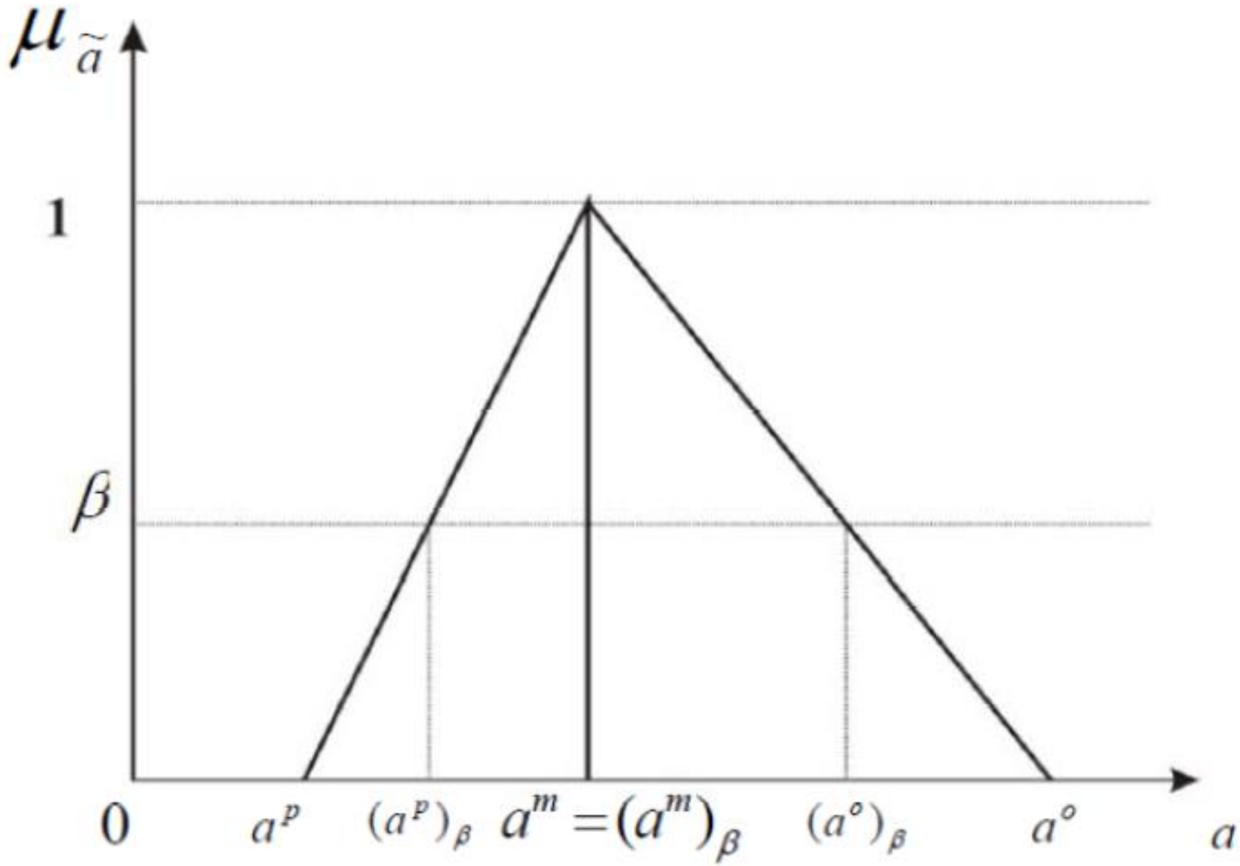
- **Value ranges of decision variables**

$$X_{ijh} \in \{0,1\}, \forall\, ij \in Z_h, \forall h \in H$$

$$Y_k \in \{0,1\}, \forall k$$

$$\alpha \geq 0$$

Uncertain parameters



Possibilistic programming

$$\begin{aligned}&\text{Max } \tilde{A}X\\ &\text{s.t. } \ \tilde{B}X \le \tilde{C}\\ &\qquad X \ge 0\end{aligned}$$

$$\begin{aligned}&\text{Min } \ (A^m - A^p)X\\ &\text{Max } \ A^m X\\ &\text{Max } \ (A^o - A^m)X\\ &\text{s.t. } \ (B^m)_\beta X \le (C^m)_\beta\\ &\qquad (B^p)_\beta X \le (C^p)_\beta\\ &\qquad (B^o)_\beta X \le (C^o)_\beta\\ &\qquad X \ge 0\end{aligned}$$

Fuzzy model

$$\text{Min } f_{1-1} = \sum_k \left[\left(a_k^m - a_k^p \right) Y_k \right]$$

$$\text{Max } f_{1-2} = \sum_k \left(a_k^m Y_k \right)$$

$$\text{Max } f_{1-3} = \sum_k \left[\left(a_k^o - a_k^m \right) Y_k \right]$$

$$\text{Min } f_2 = \alpha$$

$$\text{Min } f_3 = \sum_h \sum_{ij \in Z_h} \left(d_{ij} X_{ijh} \right)$$

S.t.

$$Y_k \le \sum_h \sum_{ij \in S_k} X_{ijh}, \ \forall k$$

$$\alpha \ge \sum_{ij \in Z_h} \left[\left(u_{ij}^m \right)_\beta X_{ijh} \right], \forall h \in H$$

$$\alpha \ge \sum_{ij \in Z_h} \left[\left(u_{ij}^p \right)_\beta X_{ijh} \right], \forall h \in H$$

$$\alpha \ge \sum_{ij \in Z_h} \left[\left(u_{ij}^o \right)_\beta X_{ijh} \right], \forall h \in H$$

$$\sum_{i \in N_j} X_{ijh} - \sum_{l \in N_j} X_{jlh} = \left\{ \begin{array}{l} -1, \ \text{if } j = s_h \\ 0, \forall j \in Z_h, j \ne s_h, t_h \\ 1, \text{if } j = t_h \end{array} \right\}, \forall h \in H$$

$$\sum_{i \in N_j} X_{ijh} \le 1, \ \forall j \ne s_h, t_h; \ \forall h \in H$$

$$\sum_{l \in N_j} X_{jlh} \le 1, \ \forall j \ne s_h, t_h; \ \forall h \in H$$

$$\sum_{ij \in Z_h} \left(d_{ij} X_{ijh} \right) \le \left(D_{max}^m \right)_\beta, \forall h \in H$$

$$\sum_{ij \in Z_h} \left(d_{ij} X_{ijh} \right) \le \left(D_{max}^p \right)_\beta, \forall h \in H$$

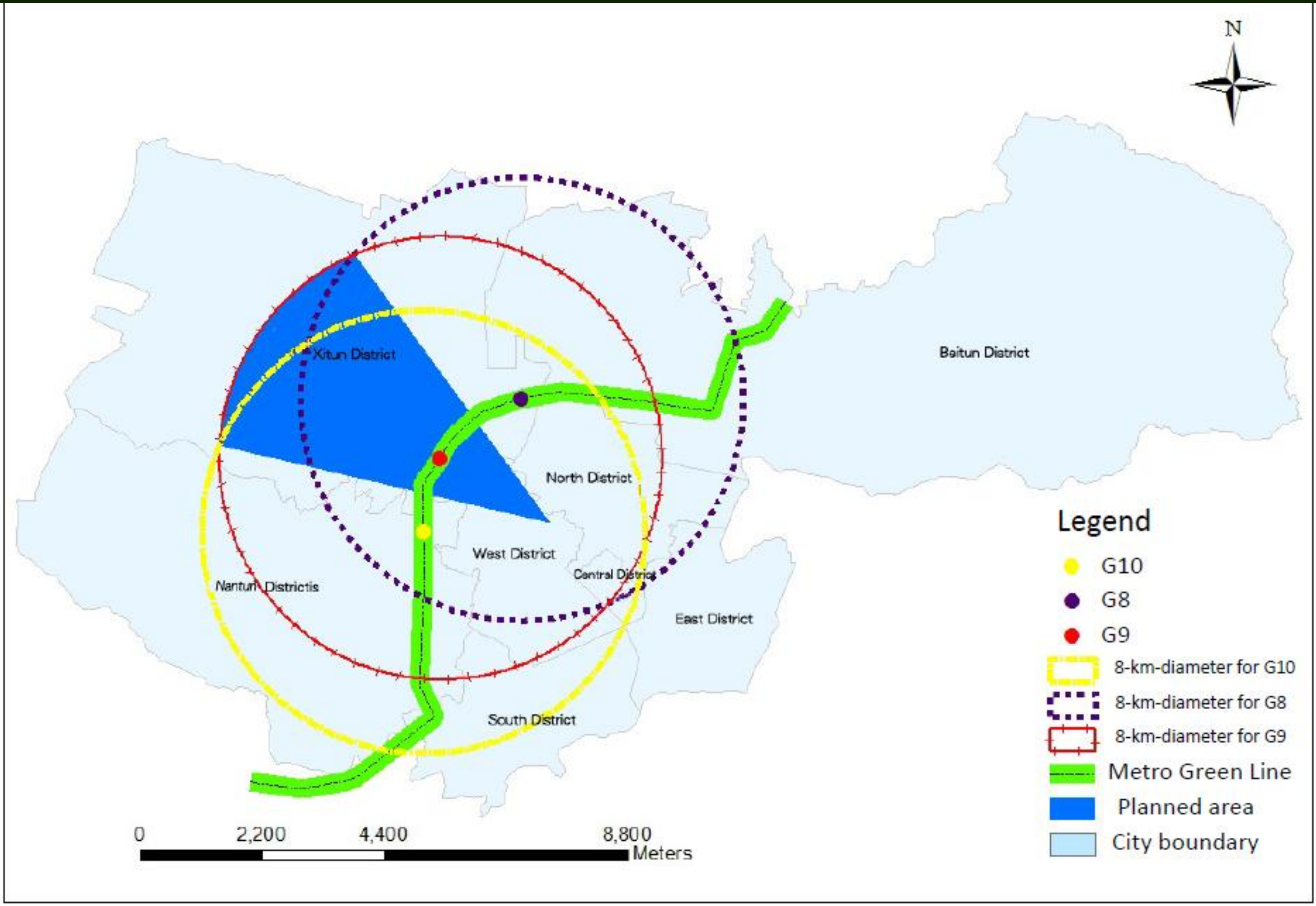
$$\sum_{ij \in Z_h} \left(d_{ij} X_{ijh} \right) \le \left(D_{max}^o \right)_\beta, \forall h \in H$$

$$X_{ijh} \in \{0,1\}, \forall ij \in Z_h, \forall h \in H$$

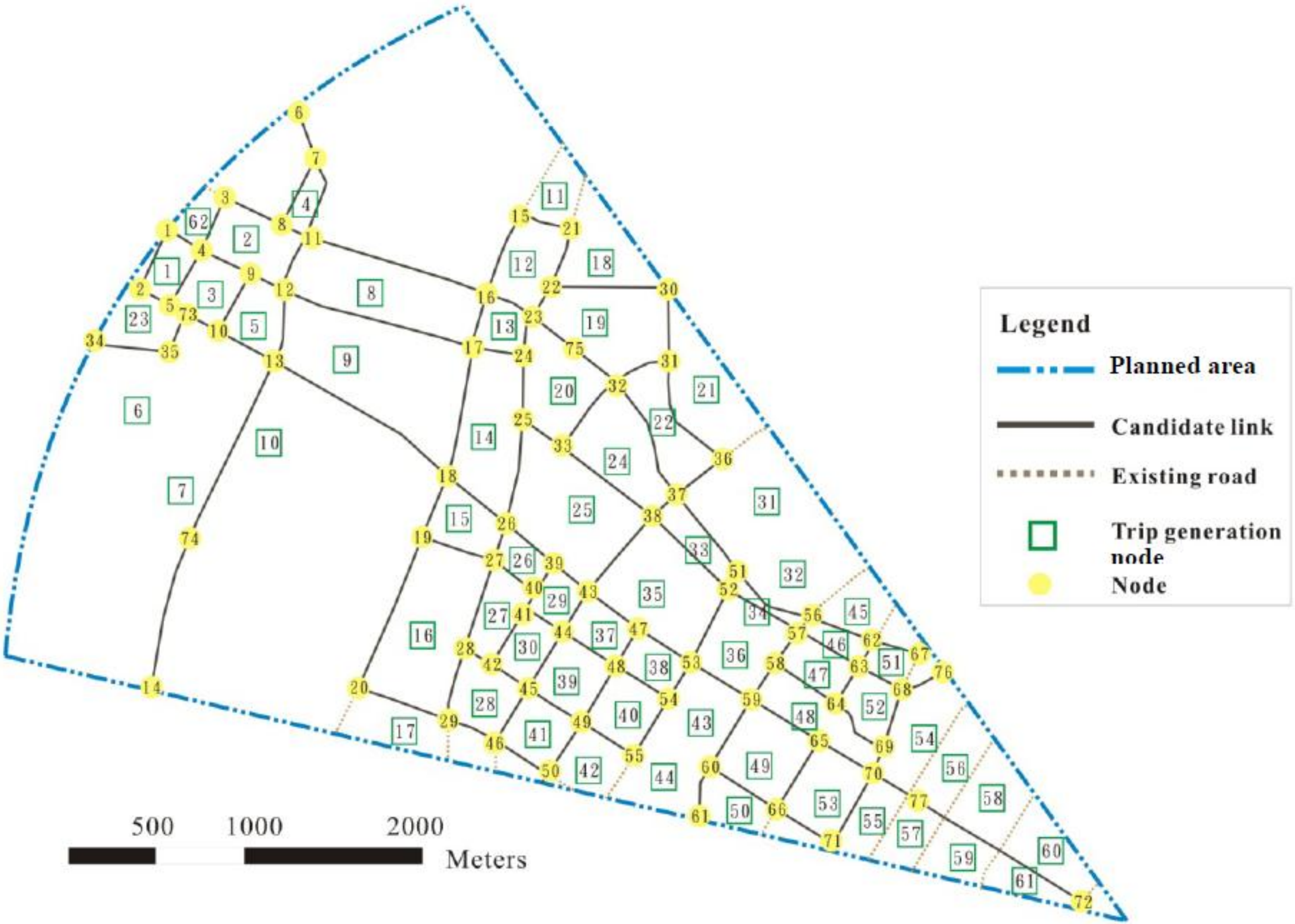
$$Y_k \in \{0,1\}, \forall k$$

$$\alpha \ge 0$$

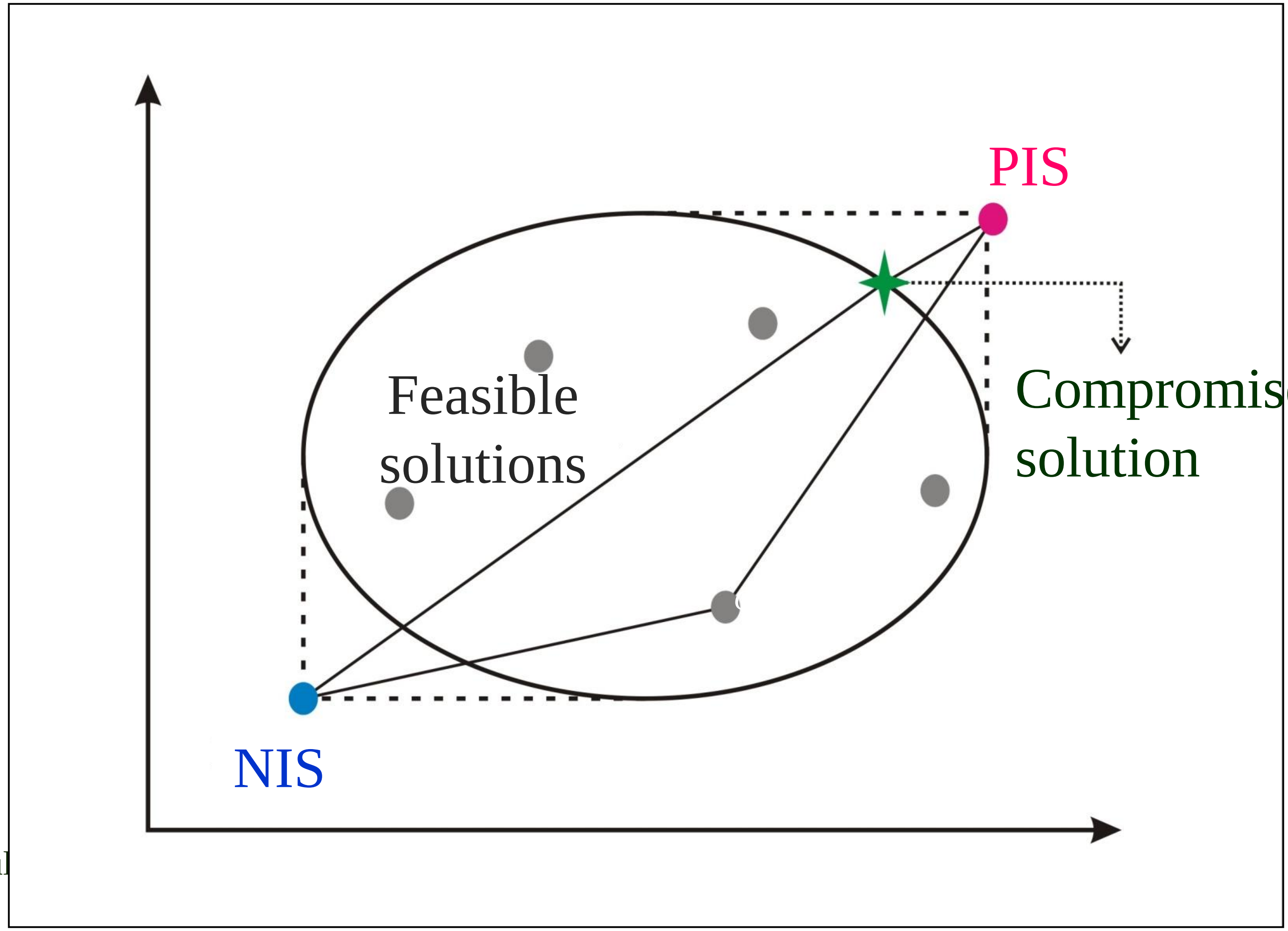
Case study: Taichung City



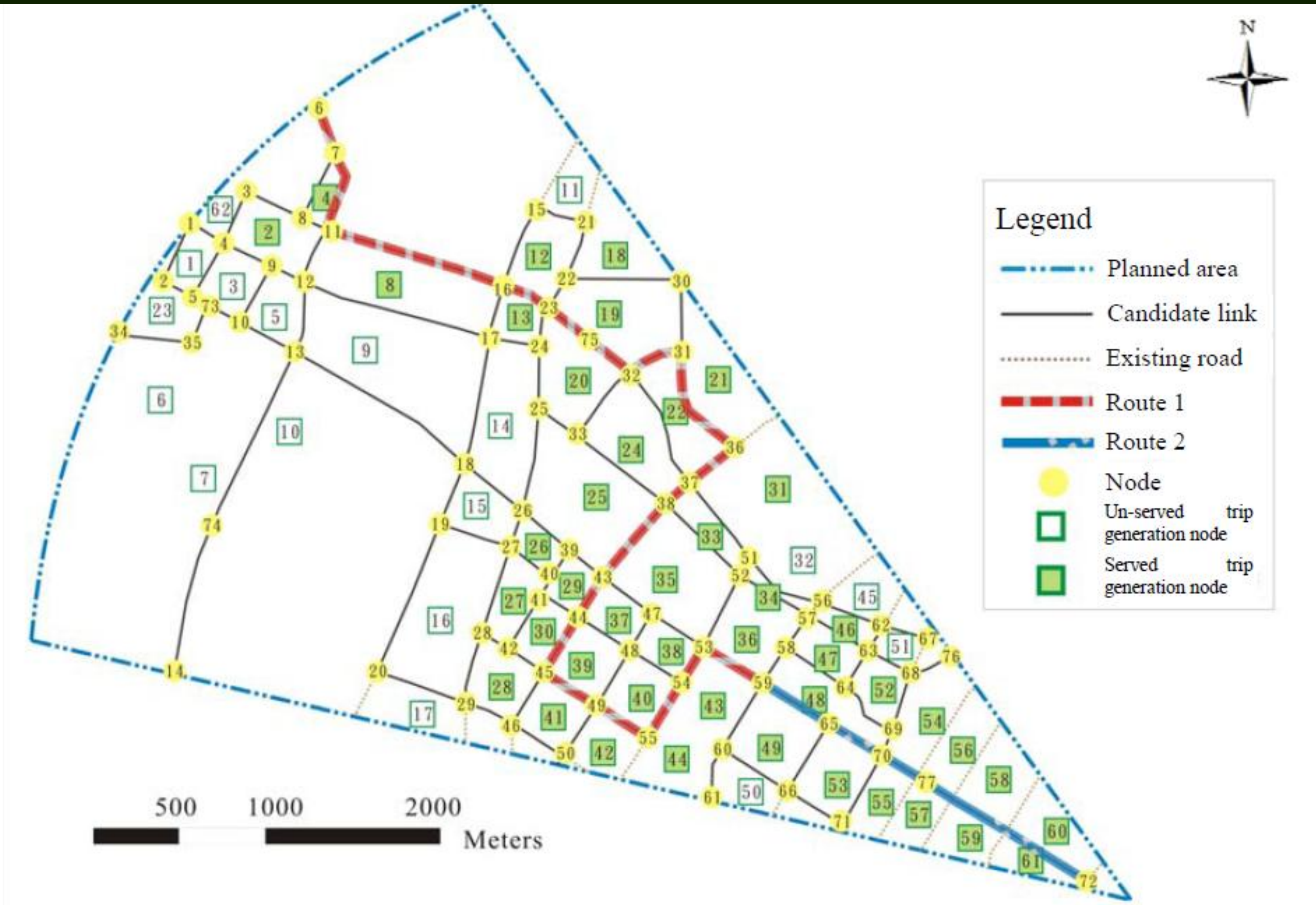
Candidate links & nodes



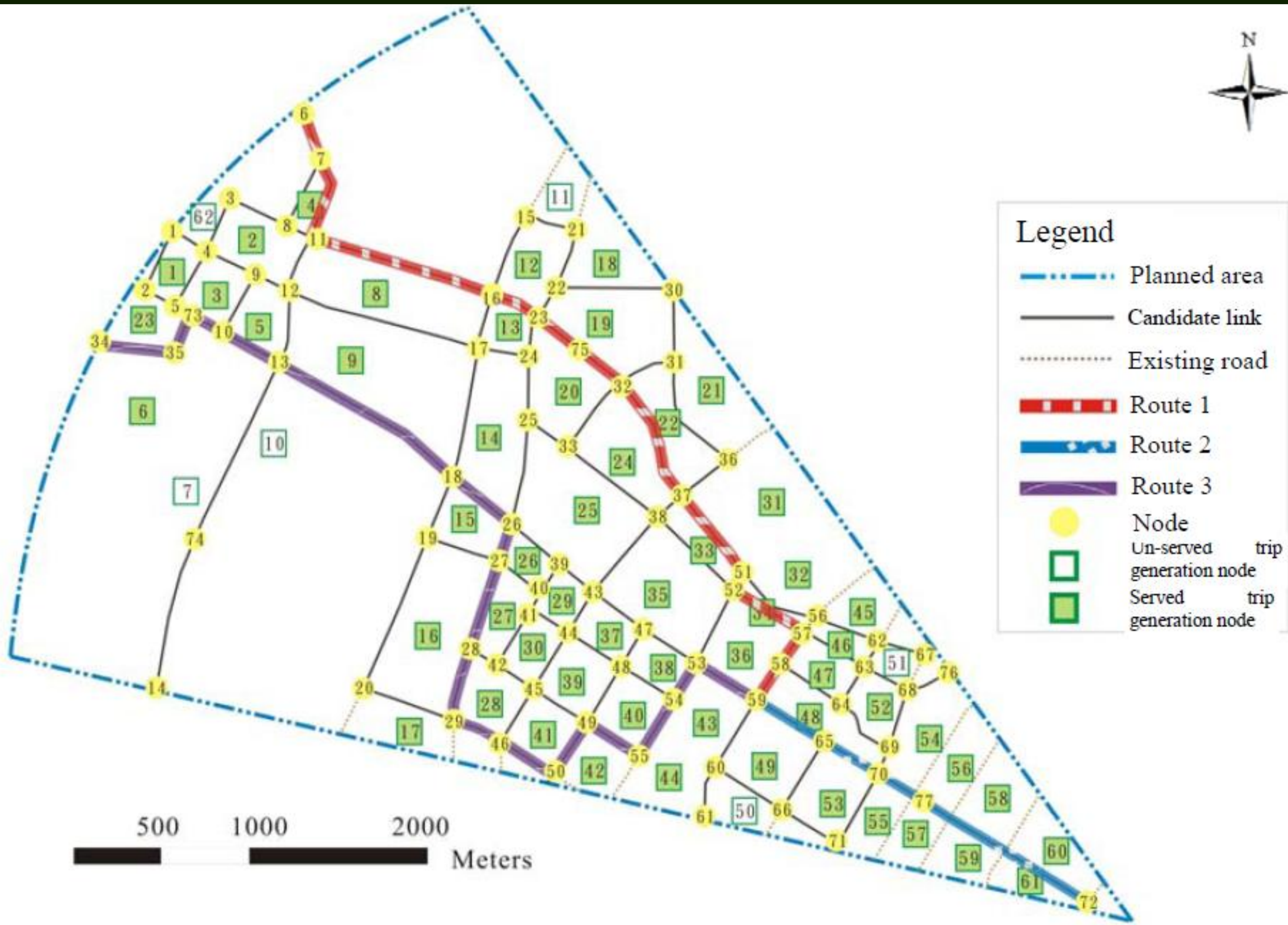
Problem solving approach: TOPSIS



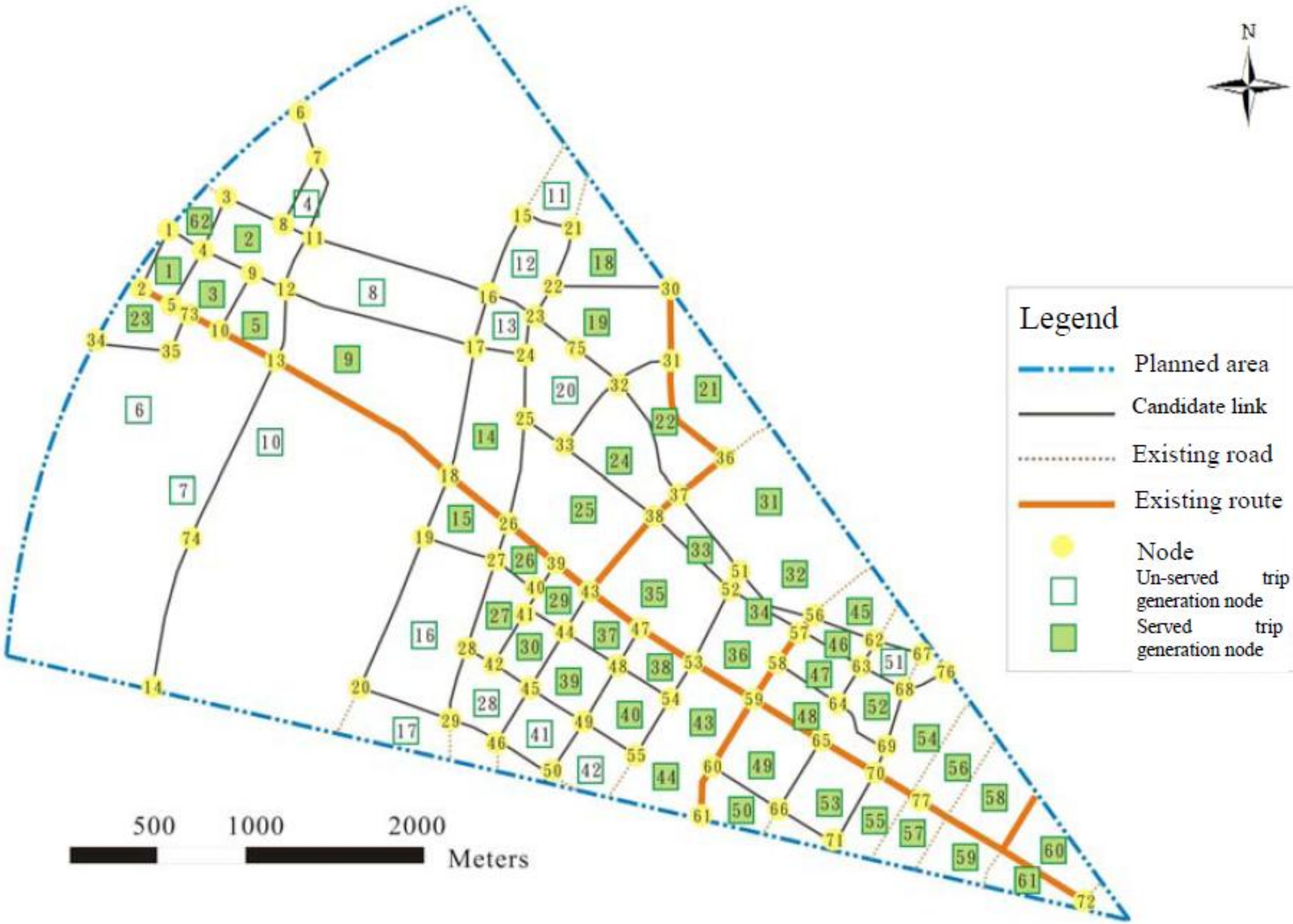
Model result: 2-route



Model result: 3-route



Existing bus routes



Objective performances

Objective	Proposed model results		Existing routes
	2-route	3-route	6-route
1, Min	670,470	927,796	474,312
	(335,235)	(463,898)	(79,052)
2, Max	1,235,116	1,698,428	862,728
	(617,558)	(566,143)	(143,788)
3, Max	1,643,149	2,233,307	1,234,986
	(821,574)	(744,436)	(205,831)
4, Min	18.18	16.11	16.37
5, Min	8,666	12,595	30,347
	(4,333)	(4,198)	(5,058)

Note: Values in the brackets are average objective values per route (original value/route number)

Comparisons

Objective	Proposed model results		Existing routes
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	(335,235)	(463,898)	(79,052)
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Note: Values in the brackets are average objective values per route (original value/route number)

- **The existing routes**
  - **Less service coverage**
  - **Longer route length**
  - **Fewer risk of coverage estimation**

- **+ route number:**
  - **+ service coverage**
  - **+ coverage estimation risk**
  - **- travel time**
  - **+ route length**

Potential contributions

- The first feeder-bus route design model that uses the multi-objective programming approach
- The model inputs can be probable ranges that help planners to handle uncertain planning conditions

Further study issues

- Linear > non-linear
- Single metro station > multiple metro stations
- Routes + stop spacing, fleet size, bus headway, service capacity