Integrating network design and line planning in rapid transit systems

CASPT12

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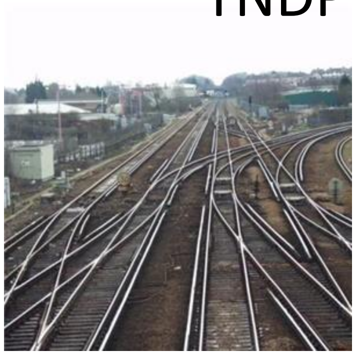
Spanish Research Grants
TRA2008-06782-C02-01/02 and TRA2011-27791-C03-01/02

Contents

- Motivation of this work
- Problem statement
- Problem formulation
- Model Solving techniques
- Computational results
- Conclusions and on-going research

TNDP bases design on *Ceder* & *Wilson* 4 phases




Global line network planning

↓

Setting Timetables → Setting frequencies / Setting departure times

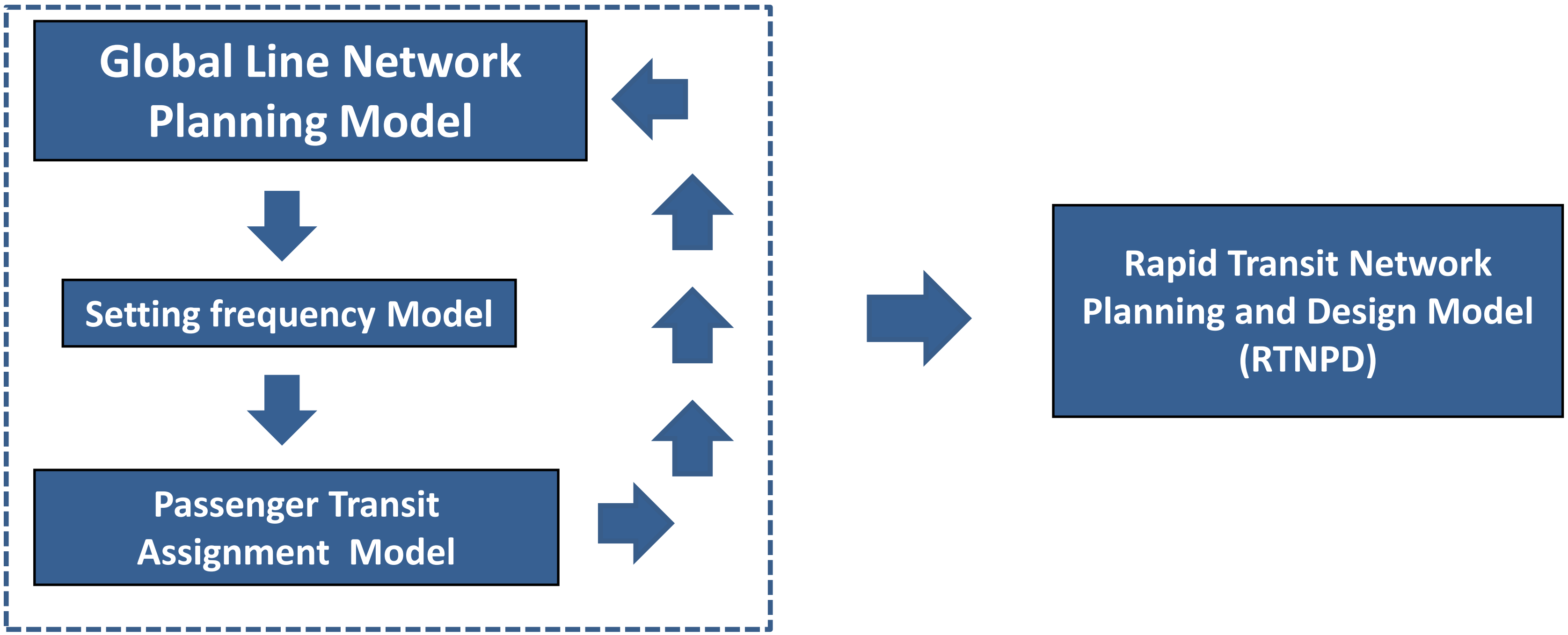
↓

Vehicle scheduling

↓

Crew scheduling

RTNPD model integrates the two first phases



Problem statement

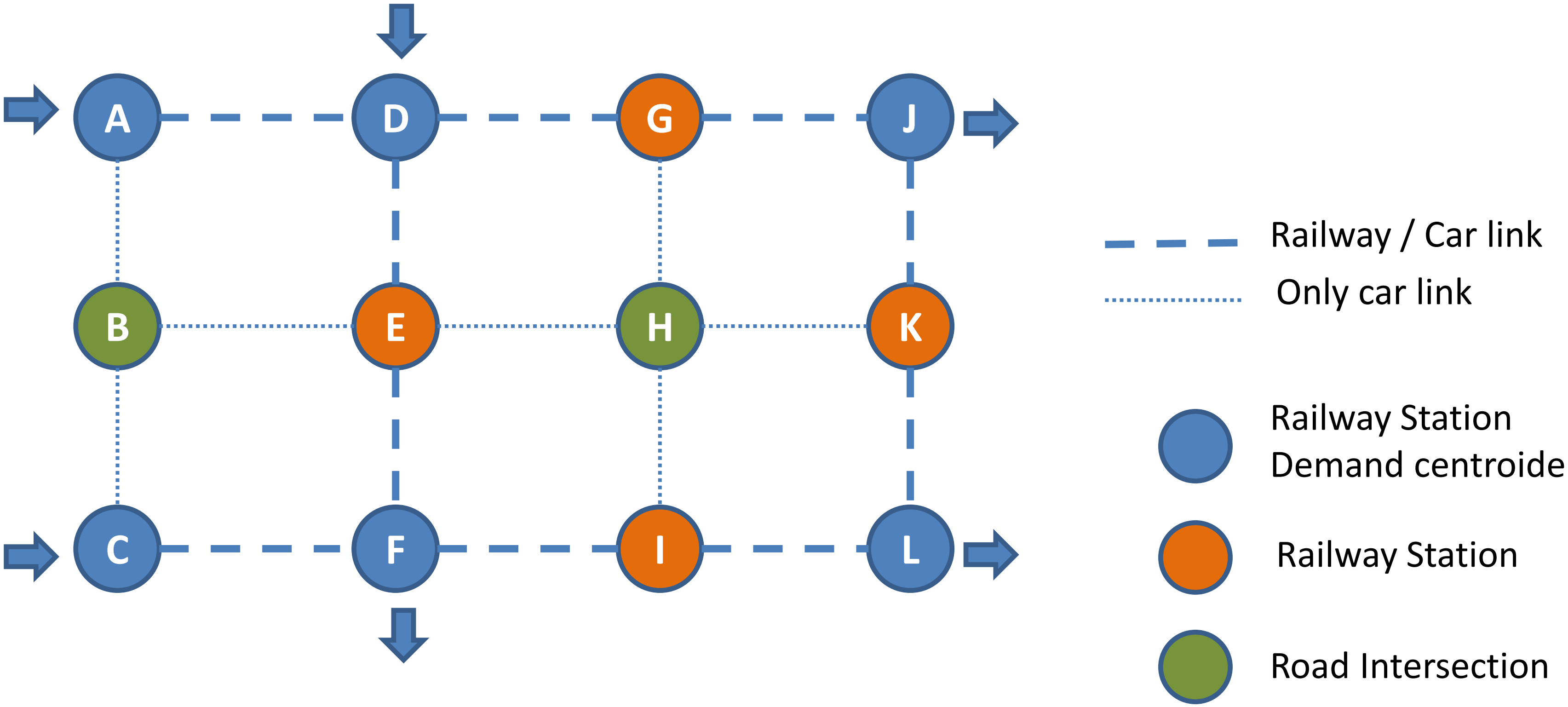
- Routing features
  - Line topology is circular and symmetrical
  - **Nodes can be considered as passing points or stops**
  - **Number of constructed stops and stretches limited by an infrastructure budget**

- Planning features
  - Homogenous vehicles fleet characteristics (Capacity)
  - Number of services are considered as continuous variables
  - **Line service limited by links capacity**
  - **Constant service time at service nodes**

- Passengers' features
  - Follow a system optimum: *User Global Time* is to be minimized
  - Its demand is known in advance and is split into O-D pairs.
  - **Passengers' service times** are considered as well as in-vehicle travel times

Features **in bold** are breakthroughs considering the current state of the art regarding to Mathematical Programming Techniques.

Network Model



Passengers' demand is split into OD pairs

Objective function

Minimize : **Pax. costs** **+** **Operator Costs**

$$Min_{[v,u,x,y,b,z]} \quad z_{pax}(v,u) \quad + \quad z_{op}(x,y,b,z)$$

Pax. costs = Travel Times + Passengers' service times at stations

$$z_{pax}(v,u) = \theta \sum_{p \in O} g_p \left[\sum_{(i,j) \in A_{COM}} t_{ij}^{COM} u_{ij}^{p} + \sum_{l \in L} \sum_{(i,j) \in A_{TP}} t_{ij}^{TP} v_{ij}^{p,l} + \sum_{l \in L} \sum_{k \in N_{TP}} \left(t_a v_{a(k)}^{p,l} + t_y v_{y(k)}^{p,l} + t_x v_{x(k)}^{p,l} \right) \right]$$

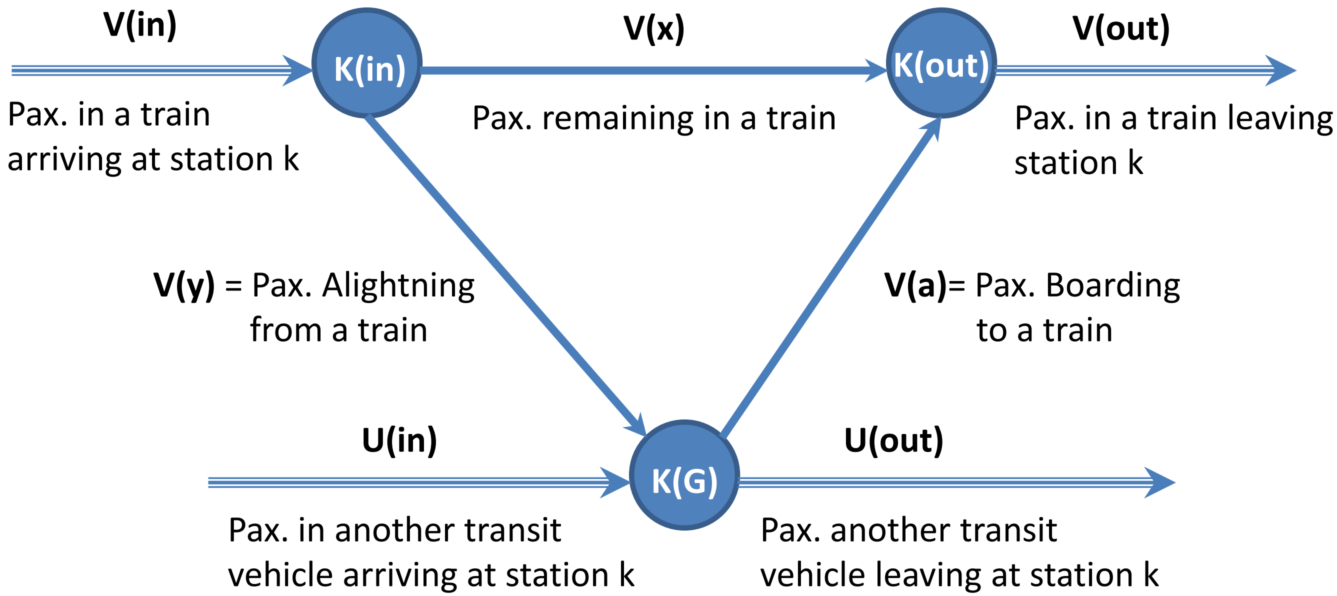
Op. costs 1 = Construction Costs + Maintenance Costs

$$z_{op}^{1}(x,y) = \sum_{i \in N_{TP}^{N}} c_i^y y_i + \sum_{(i,j) \in A_{TP}^{N}} c_{ij}^x x_{ij} + \sum_{l \in L^N} \left(\sum_{i \in N_{TP}^{N}} c_{il}^y y_i^l + \sum_{(i,j) \in A_{TP}^{N}} c_{ijl}^x x_{ij}^l \right)$$

Op. costs 2 = Assign Trains + New Trains + Services

$$z_{op}^{2}(b,z) = c_b \sum_{l \in L} b^l + c_f \Delta f_c + \sum_{l \in L_{ij}^{N}} c_{ij}^z z_{ij}^l + \sum_{l \in L_{ij}^{E}} c_{ij}^z z^l$$

A network extension allows the representation of passengers' flows at stations



$\boxed{t_y + t_a > t_x}$: Pax. from **V(in)** cannot belong to **V(y)** and **V(a)** at the same time.

Constraints

Group of Constraints	Constrained variable	Dependency
Passengers' flow balance	v(i,j,p,l), u(i,j,p), vx(i,j,p,l), vy(i,j,p,l), va(i,j,p,l)	~y(i,l), g(p).
Infrastructure budget	y(i), x(i,j)	Cmax(net), C(y,i), C(i,j,x)
Train's budget	Δf(c)	Cmax(v), C(f)
Train's fleet	b(l)	f(c,e), f(c,n), Δf(c)
Allocation of services to trains	b(l), z(l), t(l)	-
Infrastructure-to-passengers flow linking	v(i,j,p,l), vx(i,j,p,l), vy(i,j,p,l), va(i,j,p,l)	x(i,j,l), y(i,l)
Topological line design	x(i,j), y(i), x(i,j,l), y(i,l), ~y(i,l)	-
Trains' capacity	v(i,j,p,l)	q(l), z(l), z(i,j,l)
Link capacity	z(l), z(i,j,l)	/h, q(i,j), x(i,j,l)
Cycle length	t(l), x(i,j,l), ~y(i,l)	/h, PST

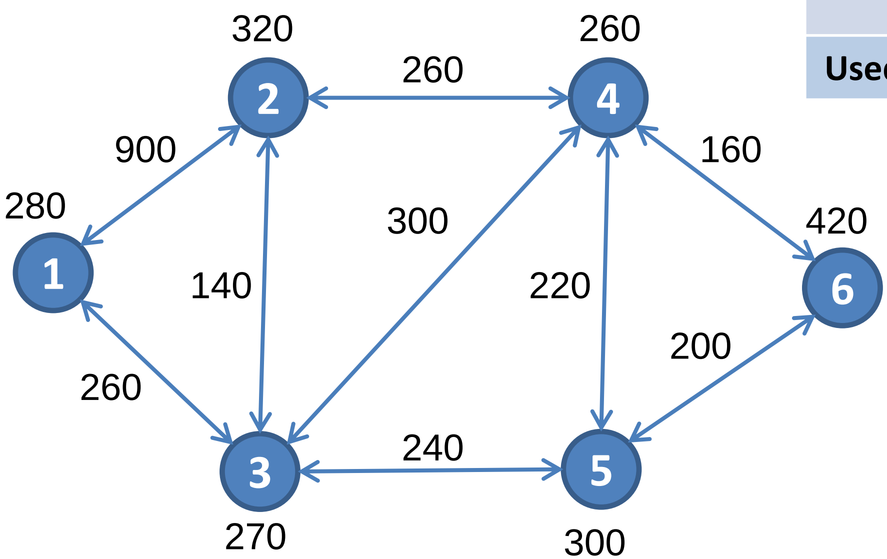
Model Solving techniques

- Exact approach
  - Branch & Bound of CPLEX 12.4.0 (**CPX B & B**)
- Heuristic approach
  - Incremental line construction procedure (**ILCP**) with,
    - Full demand assignation (**ILCPFD**)
    - Incremental demand assignation (**ILCPID**)

An illustrative example of ILCPFD procedure

Constructed lines

Line	Visited Nodes	Cycle
-	-	



L = 3
Horizon = 300 min
Cost (train) = 500 $
Cap (train) = 100 pax.
Cap (stretch) = 90 trains/ horizon

Resources

	Stations	Stretches	Trains
Available	1,2,3,4,5,6	(1,2), (1,3), (2,3), (2,4), (3,4), (3,5), (4,5), (4,6), (5,6)	10
Used	-	-	-

Budget

	Infrastructure	Trains
Available	8000 $	5000 $
Used	0	0

Demand

O	D	G(O,D)	Line
1	6	9000	-
1	5	4500	-
3	6	4500	-

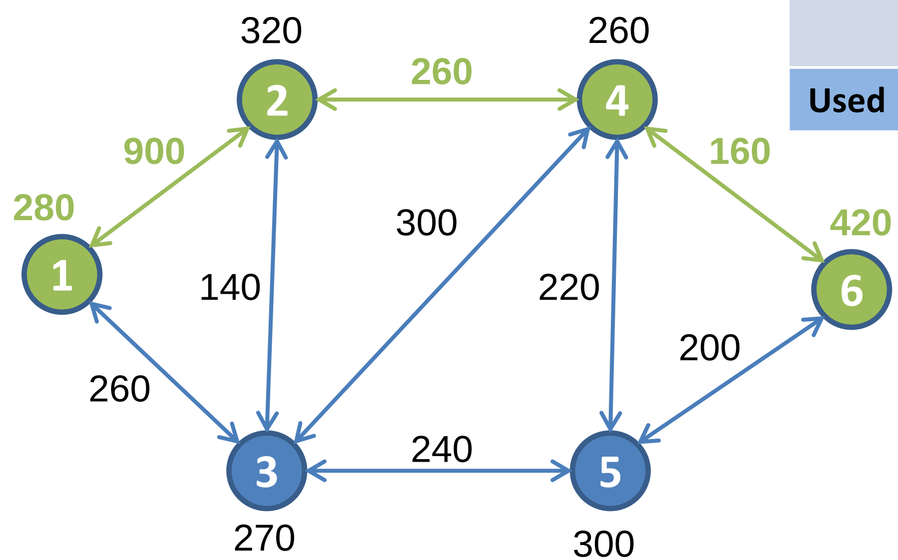
Iteration 1. A line linking the highest OD demand is constructed

Constructed lines

Line	Visited Nodes	Cycle
1	1, 2, 4 ,6	28 min

Resources

	Stations	Stretches	Trains
Available	1,2,3,4,5,6	(1,2), (1,3), (2,3), (2,4), (3,4), (3,5), (4,5), (4,6), (5,6)	10
Used	1, 6	(1,2), (2,4), (4,6)	10



Budget

	Infrastructure	Trains
Available	8000 $	5000 $
Used	2020 $	0

L = 3
Horizon = 300 min
Cost (train) = 500 $
Cap (train) = 100 pax.
Cap (stretch) = 90 trains/ horizon

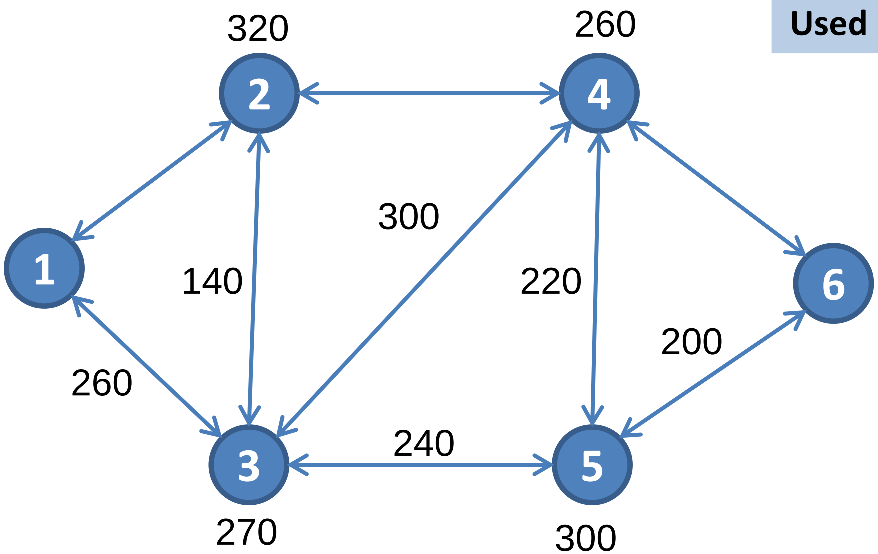
Demand

O	D	G(O,D)	Line
1	6	9000	1
1	5	4500	-
3	6	4500	-

Iteration 1 to 2. Parameters' Update

Constructed lines

Line	Visited Nodes	Cycle
1	1, 2, 4 ,6	28 min



L = 3
Horizon = 300 min
Cost (train) = 500 $
Cap (train) = 100 pax.
Cap (stretch) = 90 trains/ horizon

Resources

	Stations	Stretches	Trains
Available	2,3,4,5	(1,3), (2,3), (3,5), (4,5), (5,6)	0
Used	-	-	-

Budget

	Infrastructure	Trains
Available	5980 $	5000 $
Used	0	0

Demand

O	D	G(O,D)	Line
1	6	9000	1
1	5	4500	-
3	6	4500	-

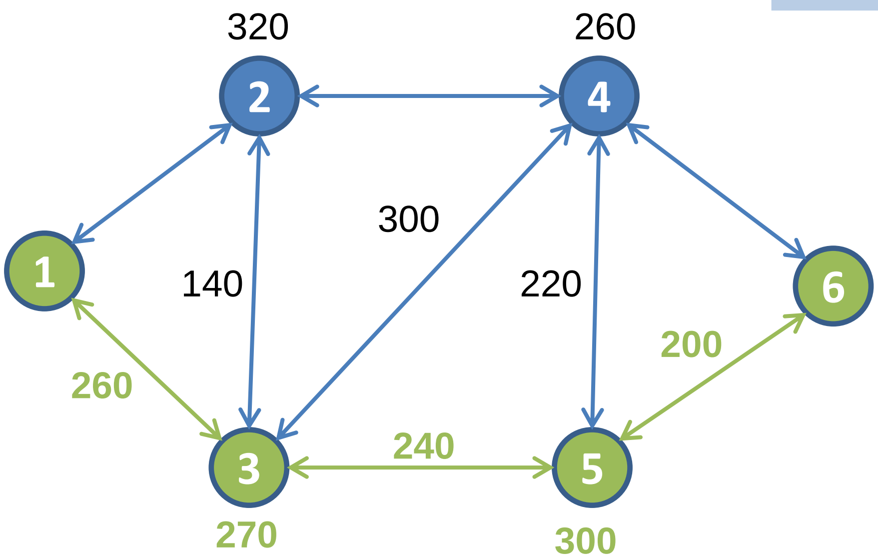
Iteration 2. Another line linking the lowest demand pairs is constructed

Constructed lines

Line	Visited Nodes	Cycle
1	1, 2, 4 ,6	28 min
2	1,3,5,6	30 min

Resources

	Stations	Stretches	Trains
Available	2,3,4,5	(1,3), (2,3), (3,4), (3,5), (4,5), (5,6)	0
Used	3,5	(1,3), (3,5), (5,6)	0



L = 3
Horizon = 300 min
Cost (train) = 500 $
Cap (train) = 100 pax.
Cap (stretch) = 90 trains / horizon

Budget

	Infrastructure	Trains
Available	5980 $	5000 $
Used	1270 $	4500 $

Demand

O	D	G(O,D)	Line
1	6	9000	1
1	5	4500	2
3	6	4500	2

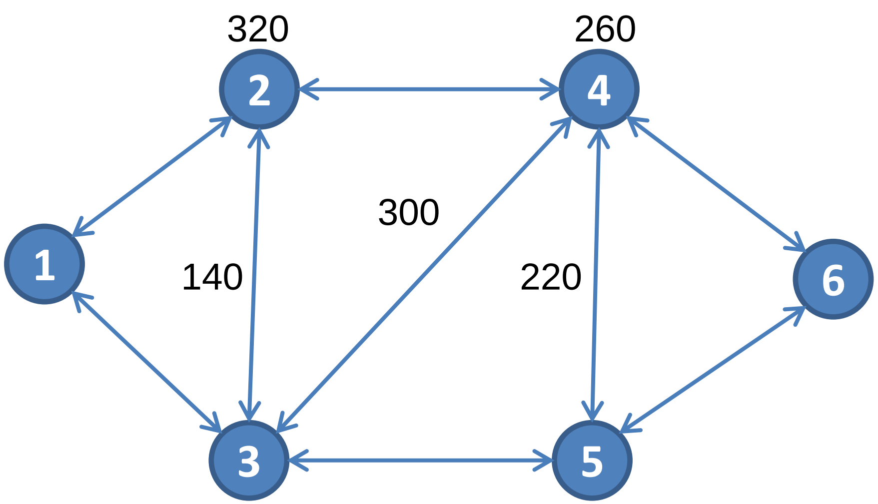
Iteration 2 to 3. Parameters' Update

Constructed lines

Line	Visited Nodes	Cycle
1	1, 2, 4 ,6	28 min
2	1,3,5,6	30 min

Resources

	Stations	Stretches	Trains
Available	2, 4	(2,3), (3,4), (4,5)	0
Used	-	-	-



L = 3
Horizon = 300 min
Cost (train) = 500 $
Cap (train) = 100 pax.
Cap (stretch) = 90 trains / horizon

Budget

	Infrastructure	Trains
Available	4710 $	500 $
Used	0	0

Demand

O	D	G(O,D)	Line
1	6	9000	1
1	5	4500	2
3	6	4500	2

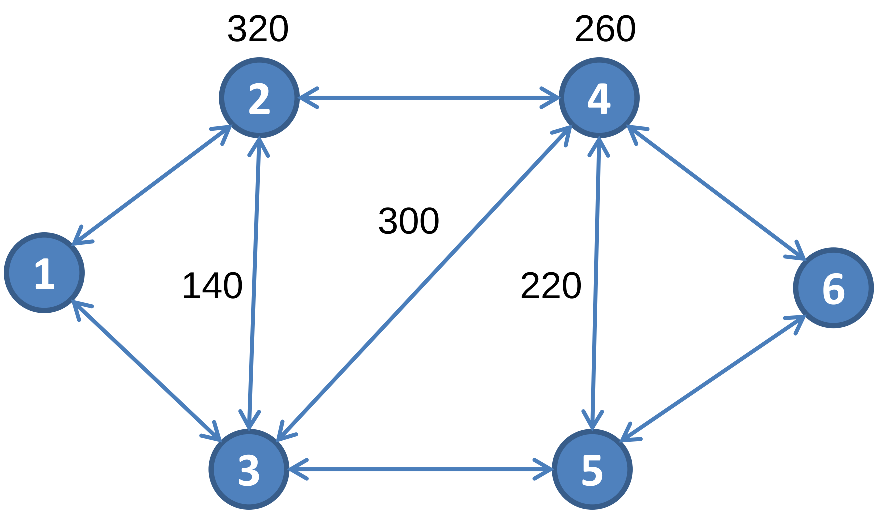
Iteration 3. No more lines are constructed

Constructed lines

Line	Visited Nodes	Cycle
1	1, 2, 4 ,6	28 min
2	1,3,5,6	30 min

Resources

	Stations	Stretches	Trains
Available	2, 4	(2,3), (3,4), (4,5)	0
Used	-	-	0



L = 3
Horizon = 300 min
Cost (train) = 500 $
Cap (train) = 100 pax.
Cap (stretch) = 9 trains / horizon

Budget

	Infrastructure	Trains
Available	4710 $	500 $
Used	0	0

Demand

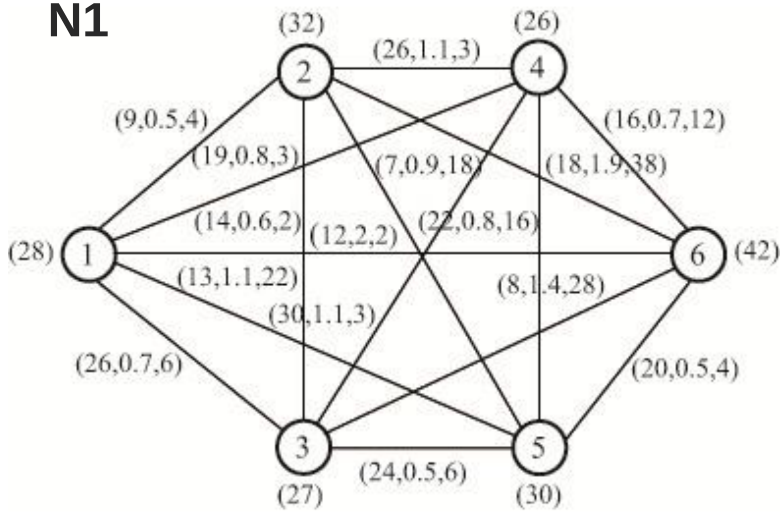
O	D	G(O,D)	Line
1	6	9000	1
1	5	4500	2
3	6	4500	2

Computational Results

- Two types of experiments carried out
  - Type 1. Model validation test
  - Type 2. Network Size vs. Performance Test

Networks for model validation test

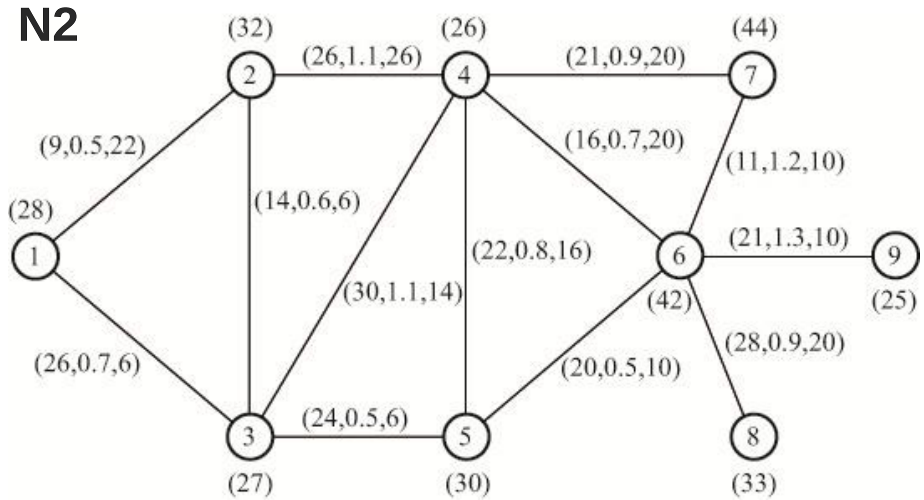
N1



		N1	N2
Nodes	N(TP)	6	9
	N(C)	6	9
Links	A(TP)	30	26
	A(C)	30	72
Demand	|W|	30	72
	G(W)	5030	10290

Table 1: Main features of the tested Networks.

N2



$$\frac{(c^{x}_{ijl}, t^{TP}_{ij}, c^{z}_{ij})}{(c^{y}_{il})} \qquad c^{x}_{ij} = 10 \cdot c^{x}_{ijl}$$

i $\quad c^{y}_{i} = 10 \cdot c^{y}_{il}$

General Results for Network N1

Exp.	|L|	η	Method	Obj.	Obj. (Pax)	T. CPU *	Gap
1	2	2.0	**ILCPID**	**7953.12**	**7518.92**	**7**	**0.14%**
			ILCPFD	**7953.12**	**7518.92**	**6**	**0.14%**
			CPX B&B	7942.31	7481.71	7	-
2	3	2.0	**ILCPID**	**8286.92**	**7777.52**	**12**	**41.6%**
			ILCPFD	5924.13	5438.13	9	1.2%
			CPX B&B	5852.85	5353.45	80	-
3	4	4.0	ILCPID	21378.74	20864.54	10	6%
			ILCPFD	20698.43	20164.03	7	2.6%
			CPX B&B	20169.86	19650.06	106	-
4	5	4.0	ILCPID	23804.90	23265.00	14	34.9%
			ILCPFD	**19361.66**	**18790.46**	**9**	**9.8%**
			CPX B&B	17640.46	17079.26	618	-
5	6	5.0	ILCPID	33176.02	32624.02	12	38%
			ILCPFD	25625.18	25048.38	9	6.6%
			CPX B&B	24037.23	23449.23	3380	-

(*) Time expressed in **seconds**

Average Gap

ILCPID = **24%**

ILCPFD = **4%**

General Results for Network N2

Exp.	|L|	η	Method	Obj.	Obj. (Pax)	T. CPU *	Gap
1	2	2.0	**ILCPID**	**24985.18**	**24522.78**	**16**	**< 0.01%**
			ILCPFD	**24997.66**	**24532.06**	**12**	**0.05%**
			CPX B&B	24984.01	24521.61	36	-
2	3	2.0	ILCPID	21231.80	20757.00	16	7.96%
			ILCPFD	22038.79	21540.59	16	12.07%
			CPX B&B	19665.96	19170.96	191	-
3	4	3.0	ILCPID	44983.20	44481.40	18	17.82%
			ILCPFD	42065.18	41564.38	15	10.17%
			CPX B&B	38180.48	37681.08	271	-
4	5	4.0	ILCPID	68381.00	67877.60	23	19.81%
			ILCPFD	**66176.45**	**65673.55**	**19**	**15.94%**
			CPX B&B	57079.09	56557.09	1560	-
5	6	5.0	**ILCPID**	**94707.50**	**94199.91**	**24**	**20.76%**
			ILCPFD	81378.68	80797.58	22	3.76%
			CPX B&B	78429.04	77823.04	2255	-

Average Gap

ILCPID = **13%**

ILCPFD = **8%**

(*) Time expressed in **seconds**

Detailed Results for networks N1 & N2

Network N1 Results

Exp.	|L|	NV	NZ	Uavg (l)	Umax (l)	Uavg (v)	Umax (v)
1	4	4	8	34.7%	66.7%	89.8%	100%
2	5	5	10	50.7%	100%	89.8%	100%
3	6	6	12	53 %	100%	93%	100%
4	7	7	14	60.6%	100%	93.43%	100%
5	8	8	16	66.7%	100%	94.1%	100%

|L| = Number of lines constructed
NV = Number of used trains
NZ = Number of services performed

Measures

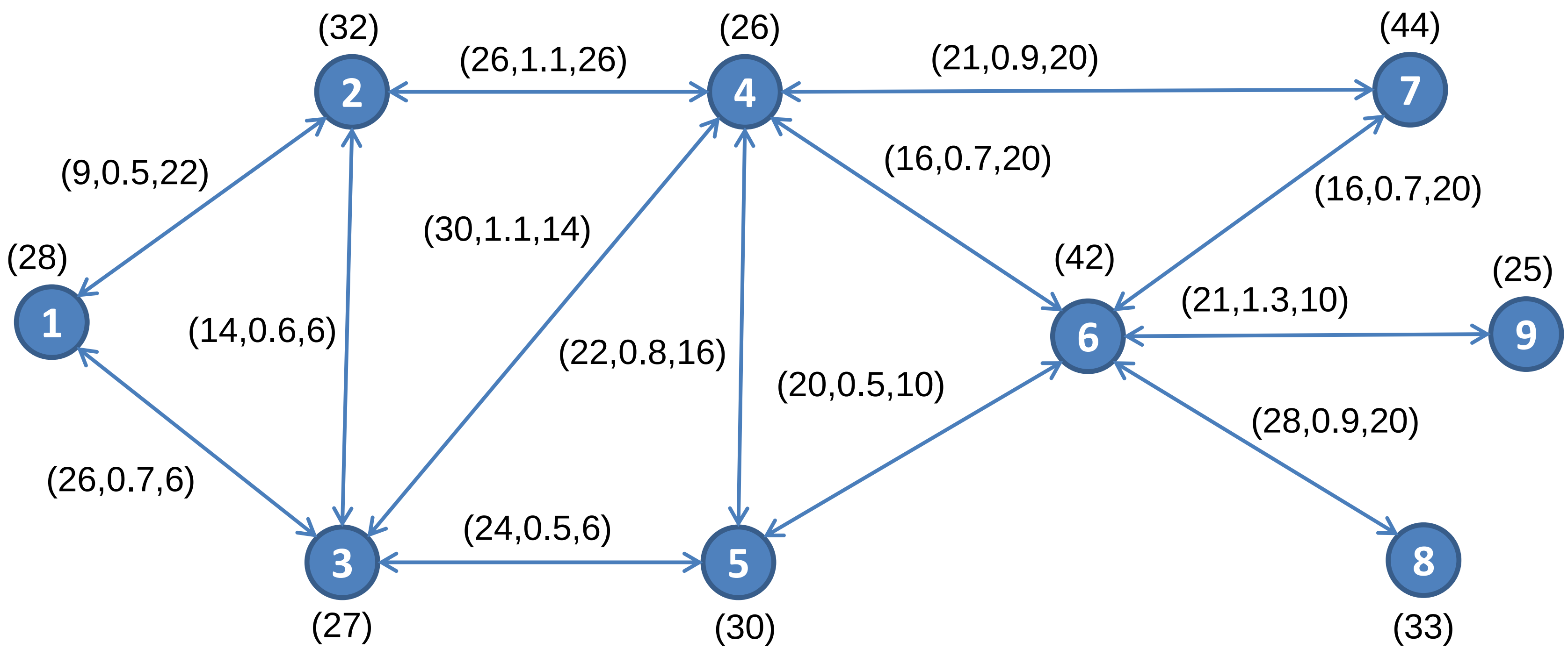
Uavg (l) = Average line utilization
Umax (l) = Maximum line utilization
Uavg (v) = Average vehicle utilization
Umax(v) = Maximum vehicle utilization

The model tends to fill vehicles as much as possible so that less services are required !

Network N2 Results

Exp.	|L|	NV	NZ	Uavg (l)	Umax (l)	Uavg (v)	Umax (v)
1	4	4	8	34.7%	66.7%	89.8%	100%
2	5	5	10	50.7%	100%	89.8%	100%
3	6	6	12	53 %	100%	93%	100%
4	7	7	14	60.6%	100%	93.43%	100%
5	8	8	16	66.7%	100%	94.1%	100%

N2 is used as initial network for the Network Size vs. Performance Test



General Results for the Network Size vs. Performance Test

Exp	N	A	Obj.	Obj (Pax)	T. CPU *
1	9	72	32040.91	31606.11	11
2	10	76	28530.55	28107.45	14
3	11	80	28530.55	28107.45	20
4	12	84	26413.91	25979.21	17
5	13	88	24285.54	23848.04	22
6	14	92	24285.54	23848.04	24
7	15	96	21259.58	20836.18	32
8	16	100	21259.58	20836.18	39
9	17	104	19263.35	18850.15	38
10	18	108	19263.35	18850.15	31
11	19	112	19263.35	18850.15	165
12	20	116	18845.09	18411.49	147
13	21	120	17727.06	17291.86	384
14	22	124	17727.06	17291.86	762
15	**23**	**128**	**17727.06**	**17291.86**	**861**
16	**24**	**132**	**17429.95**	**17003.65**	**28703**

Computational time exploits!

(*) Time expressed in **seconds**, **|L| = 1, |W| = 72**

Conclusions

- A network design and line planning model has been presented for modelling rapid transit systems
- The network design determines the extension of the current set of working lines,
  - By means of a set of candidates stations
  - Without exceeding the available network infrastructure budget
- The line planning assigns vehicles and services while meeting
  - Link and vehicle capacity constraints
  - Vehicle's fleet maximum size
  - Planning horizon requirements
- Express / point-to point lines can be constructed thanks to
  - The consideration of passengers' service times at stations
  - The role determination of the line stations as passing points or service points
- The model is formulated by means of mixed integer linear programming and it is split heuristically into a series of subproblems with the same mathematical structure to solve efficiently small-sized networks

Ongoing Research

- An alternative decomposition approach to speed-up the model resolution
  - ➢ Benders with convergence enhancements (Papadakos 2008)
  - ➢ Ad hoc methods for solving the master problem
- Inclusion of a transportation mode choice model based on passengers utility functions
  - ➢ Demand mode splitting constraints (Marin & Ródenas 2008)
- Consideration of passengers' strategies instead of System Optimum

THANKS FOR YOUR ATTENTION

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Appendix I

Detailed formulation of Model’s constraints:

- Passengers flow balance constraints
- Budget and vehicle fleet Constraints
- Allocation of services to vehicles constraints
- Infrastructure-to-passengers flow linking constraints
- Topological line design constraints
- Vehicles capacity constraints
- Link capacity constraints
- Cycle length constraints

Passengers flow balance constraints

$$\sum_{l\in L}\sum_{j\in A_{TP}(i)} v_{ij}^{p,l} - \sum_{l\in L}\sum_{j\in A_{TP}(i)} v_{ji}^{p,l} + \sum_{j\in A_{COM}(i)} u_{ij}^{p} - \sum_{j\in A_{COM}(i)} u_{ji}^{p} = t_i^w, \quad \forall p\in O, i\in N$$

$$\sum_{j\in A_{TP}(i)} v_{ji}^{p,l} = v_{y(i)}^{p,l} + v_{x(i)}^{p,l}, \quad \forall i\in N_{TP}^{E}(l), p\in O, l\in L^E$$

$$\sum_{j\in A_{TP}(i)} v_{ij}^{p,l} = v_{a(i)}^{p,l} + v_{x(i)}^{p,l}, \quad \forall i\in N_{TP}^{E}(l), p\in O, l\in L^E$$

$$v_{y(i)}^{p,l} + v_{x(i)}^{p,l} + \left(1-\overline{y}_i^{\,l}\right) \geq \sum_{j\in A_{TP}(i)} v_{ji}^{p,l} \geq v_{y(i)}^{p,l} + v_{x(i)}^{p,l} + \left(\overline{y}_i^{\,l} - 1\right), \quad \forall i\in N_{TP}^{N}, p\in O, l\in L^N$$

$$v_{a(i)}^{p,l} + v_{x(i)}^{p,l} + \left(1-\overline{y}_i^{\,l}\right) \geq \sum_{j\in A_{TP}(i)} v_{ij}^{p,l} \geq v_{a(i)}^{p,l} + v_{x(i)}^{p,l} + \left(\overline{y}_i^{\,l} - 1\right), \quad \forall i\in N_{TP}^{N}, p\in O, l\in L^N$$

Where,

$$t_i^w = \begin{cases} 1 & \text{if} \quad i = p \\ -\dfrac{g_{pi}}{g_p} & \text{if} \quad i \neq p, (p,i)\in W \\ 0 & \text{if} \quad (p,i)\notin W \end{cases}$$

Budget and vehicle fleet Constraints

$$\sum_{i \in N_{TP}^{N}} c_i^y y_i + \sum_{(i,j) \in A_{TP}^{N}} c_{ij}^x x_{ij} \le c_{\max}^{Net}$$

$$0 \le \Delta f_c \le \left\lfloor \frac{c_{\max}^{veh}}{c_f} \right\rfloor$$

$$\sum_{l \in L} b^l \le f_c^E + f_c^N + \Delta f_c$$

Input parameters

$c_{\max}^{net}, c_{\max}^{veh}$: Available budgets for infrastructure and acquiring new vehicles

c_i^y, c_{ij}^x : Costs of constructing a station i and stretch (i,j)

f_c^E, f_c^N: Fleet of working and unused vehicles.

Allocation of services to vehicles constraints

$$\boxed{b^l = t^l \cdot z^l, \quad l \in L}$$ ⇨ Non linear!

But it can be linearized by means of the following procedure:

1) Discretization of *t(l)* variables:

$$b_i^l = t_i \cdot \delta_i^l \cdot z^l, \quad \forall i \in T, l \in L$$

$$b^l = \sum_{i \in T} b_i^l, \quad \forall l \in L$$

$$t^l = \sum_{i \in T} \delta_i^l \cdot t_i, \quad \forall l \in L$$

$$\sum_{i \in T} \delta_i^l \leq 1, \quad \forall l \in L$$

2) Linearization of the product of binary *δ(i,l)* and continuous *z(l)* variables

(Groover 1975 Section 2)

$$0 \leq b_i^l \leq t_i \cdot z_{\max} \cdot \delta_i^l, \qquad \forall i \in T, l \in L$$

$$t_i \cdot \left[z^l - z_{\max}\left(1 - \delta_i^l\right)\right] \leq b_i^l \leq t_i \cdot z^l, \quad \forall i \in T, l \in L$$

Infrastructure-to-passengers flow linking constraints

$$\sum_{p\in O}\left(v_{ij}^{p,l}+v_{ij}^{p,l}\right)\leq x_{ij}^{l},\quad \forall (i,j)\in A_{TP}^{N}, i<j, l\in L^{N}$$

$$\sum_{j\in E_{TP}(i)} v_{ij}^{p,l}-\sum_{j\in I_{TP}(i)} v_{ji}^{p,l}\leq \overline{y}_{i}^{l},\quad \forall i\in N_{TP}^{N}, p\in O, l\in L^{N}$$

$$\sum_{j\in I_{TP}(i)} v_{ji}^{p,l}-\sum_{j\in E_{TP}(i)} v_{ij}^{p,l}\leq \overline{y}_{i}^{l},\quad \forall i\in N_{TP}^{N}, p\in O, l\in L^{N}$$

$$\sum_{p\in O}\left(v_{x(i)}^{p,l}+v_{y(i)}^{p,l}+v_{a(i)}^{p,l}\right)\leq M\cdot\overline{y}_{i}^{l},\quad \forall i\in N_{TP}^{N}, l\in L^{N}$$

Where,

$$M=\left|O\right|$$

Topological line design constraints

$$\sum_{l \in L^N} \overline{y}_i^l \le |L^N| \cdot y_i, \quad \forall i \in N_{TP}^N$$

$$\sum_{l \in L^N} x_{ij}^l \le \left|L^N\right| \cdot x_{ij}, \quad \forall (i,j) \in A_{TP}^N, i < j$$

$$\sum_{i \in N_{TP}^N} \overline{y}_i^l \ge 2 \cdot \sum_{i \in T} \delta_i^l, \quad \forall l \in L^N$$

$$\overline{y}_i^l \le y_i^l, \quad \forall i \in N_{TP}^N, l \in L^N$$

$$y_i^l \le \sum_{j \in E_{TP}^N(i)} x_{ij}^l + \sum_{j \in I_{TP}^N(i)} x_{ji}^l \le 2 \cdot y_i^l, \quad \forall i \in N_{TP}^N, l \in L^N$$

$$\sum_{i \in T} \delta_i^l + \sum_{(i,j) \in A_{TP}^N} x_{ij}^l = \sum_{i \in N_{TP}^N} y_i^l, \quad \forall l \in L^N$$

$$\sum_{i \in T} \delta_i^l \le \sum_{(i,j) \in A_{TP}^N} x_{ij}^l \le 0.5 \cdot \left|A_{TP}^N\right| \cdot \sum_{i \in T} \delta_i^l, \quad \forall l \in L^N$$

$$\sum_{i \in Q} \sum_{j \in Q} x_{ij}^l \le |Q| - 1, \quad Q \subset N_{TP}^N, |Q| \ge 3, l \in L^N$$

Vehicles capacity constraints

$$\sum_{p \in O}\left(g_p \cdot v_{ij}^{p,l}\right) \leq q^l \cdot z^l, \quad \forall (i,j) \in A_{TP}^{E}(l), l \in L^E$$

$$\sum_{p \in O}\left(g_p \cdot v_{ij}^{p,l}\right) \leq q^l \cdot z_{ij}^l, \quad \forall (i,j) \in A_{TP}^{N}, i < j, l \in L^N$$

$$\sum_{p \in O}\left(g_p \cdot v_{ij}^{p,l}\right) \leq q^l \cdot z_{ji}^l, \quad \forall (i,j) \in A_{TP}^{N}, i > j, \forall l \in L^N$$

Input parameters

q^l: Capacity of a vehicle assigned to line l.

Link capacity constraints

$$\sum_{l \in L_{ij}^{E}} z^{l} + \sum_{l \in L_{ij}^{N}} z_{ij}^{l} \leq q_{ij} \cdot H, \quad \forall (i, j) \in A_{TP}, i < j$$

$$\boxed{z^{l} \cdot x_{ij}^{l} = z_{ij}^{l}, \quad \forall (i, j) \in A_{TP}^{N}, l \in L^{N}} \Rightarrow \text{Non linear!}$$

Linearization of the product of binary *x(i,j,l)* and continuous *z(l)* variables by means of **Groover 1975 Section 2**, resulting in:

$$\boxed{\begin{aligned}
& z_{ij}^{l} \leq z^{l}, \quad \forall (i, j) \in A_{TP}^{N}, i < j, l \in L^{N} \\
& z_{ij}^{l} \leq z_{\max} \cdot x_{ij}^{l}, \quad \forall (i, j) \in A_{TP}^{N}, i < j, l \in L^{N} \\
& z^{l} - z_{ij}^{l} \leq \left(1 - x_{ij}^{l}\right) \cdot z_{\max}, \quad \forall (i, j) \in A_{TP}^{N}, i < j, l \in L^{N} \\
& z^{l} \leq z_{\max} \cdot \sum_{(i, j) \in A_{TP}^{N}, i < j} x_{ij}^{l}, \quad \forall l \in L^{N}
\end{aligned}}$$

Cycle length constraints

$$\bar{h} \cdot t^{l} \geq \sum_{(i,j) \in l_{ij}} \left(\frac{d_{ij}^{TP}}{\lambda_{ij}} + \frac{d_{ji}^{TP}}{\lambda_{ji}} \right) + 2 \cdot PST \cdot \left(\left| N_{TP}^{E}(l) \right| - 1 \right), \quad \forall l \in L^{E}$$

$$\bar{h} \cdot t^{l} \geq \sum_{(i,j) \in A_{TP}^{N}} x_{ij}^{l} \cdot \left(\frac{d_{ij}^{TP}}{\lambda_{ij}} + \frac{d_{ji}^{TP}}{\lambda_{ji}} \right) + 2 \cdot PST \cdot \left(\sum_{i \in N_{TP}^{N}} \bar{y}_{i}^{l} - 1 \right), \quad \forall l \in L^{N}$$

Where,

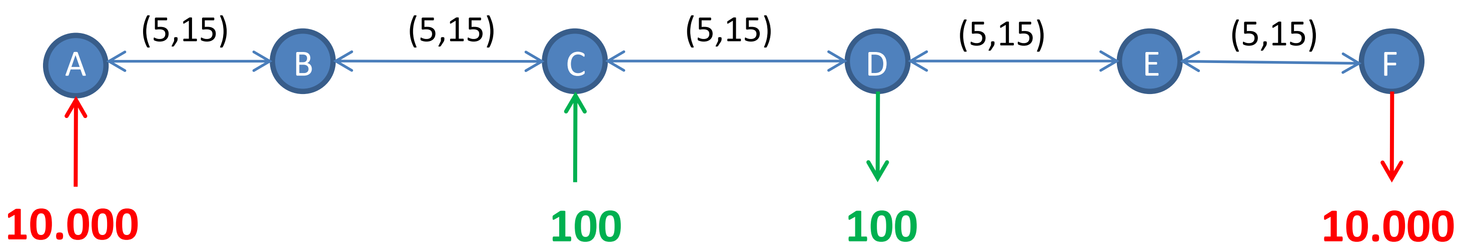
$$\bar{q}_{ij} = \frac{q_{ij} + q_{ji}}{2}, \quad \forall (i,j) \in A_{TP}, i < j$$

Appendix II

The model's working

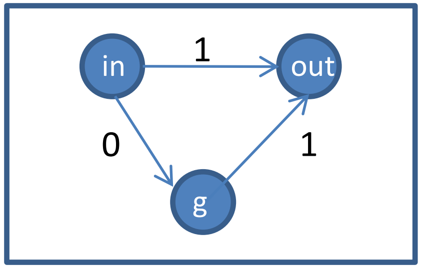
An illustrative example to show the model's working

A rectilinear transit network with 6 stations , 10 stretches and 2 OD-pairs.



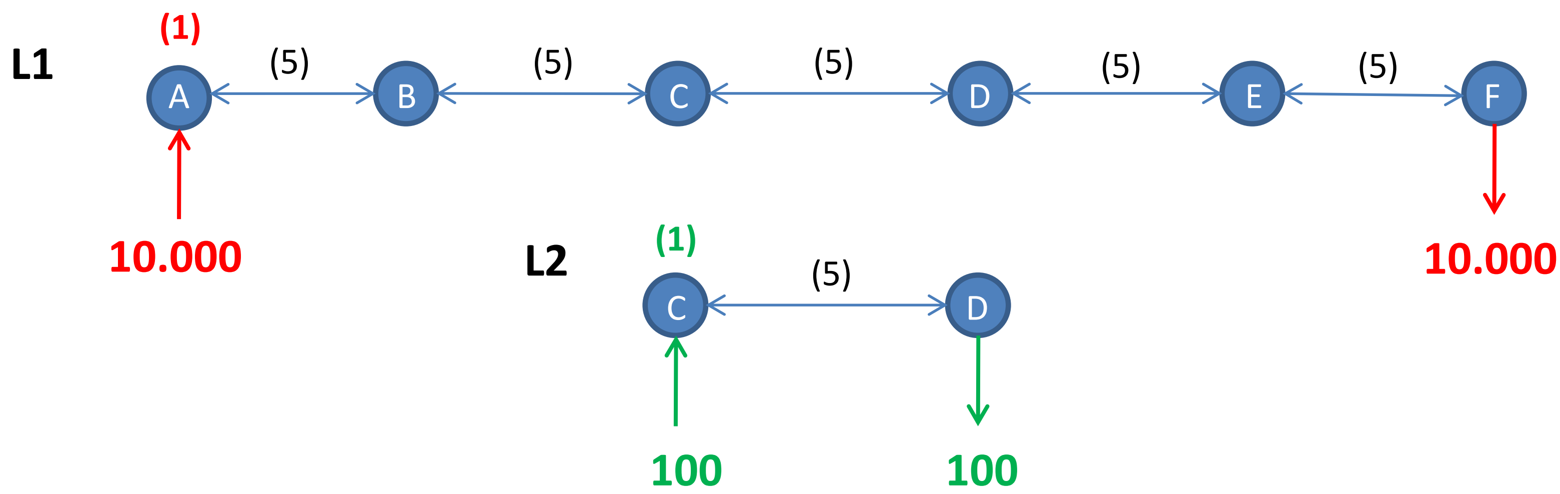
Suppose that,

- We want to construct up to **2 lines**
- Maintenance costs are **negligible**
- We have an **unlimited budget and time horizon**
- But exchange times at station are as follows

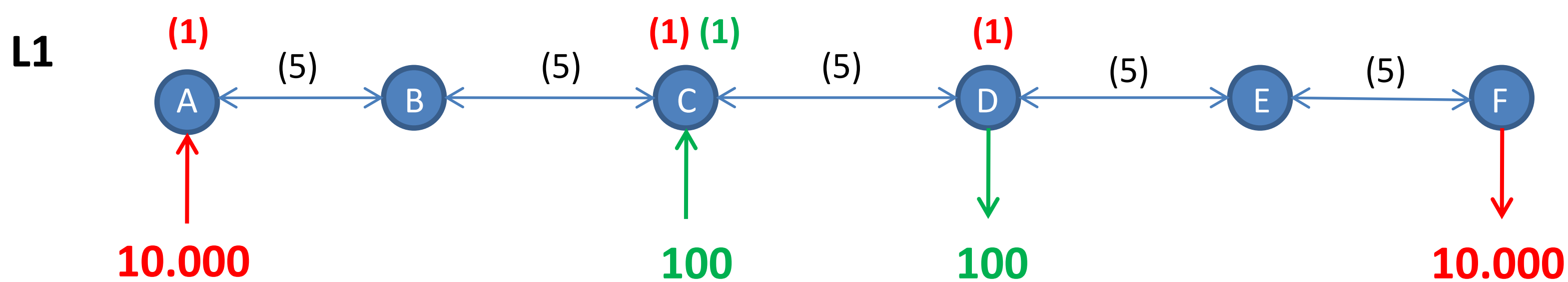


The network flow extension allows building point-to-point lines

Using our model: Z(pax) = (25 + **1**) x 10.000 + (5 + 1) x 100 = **260.600**



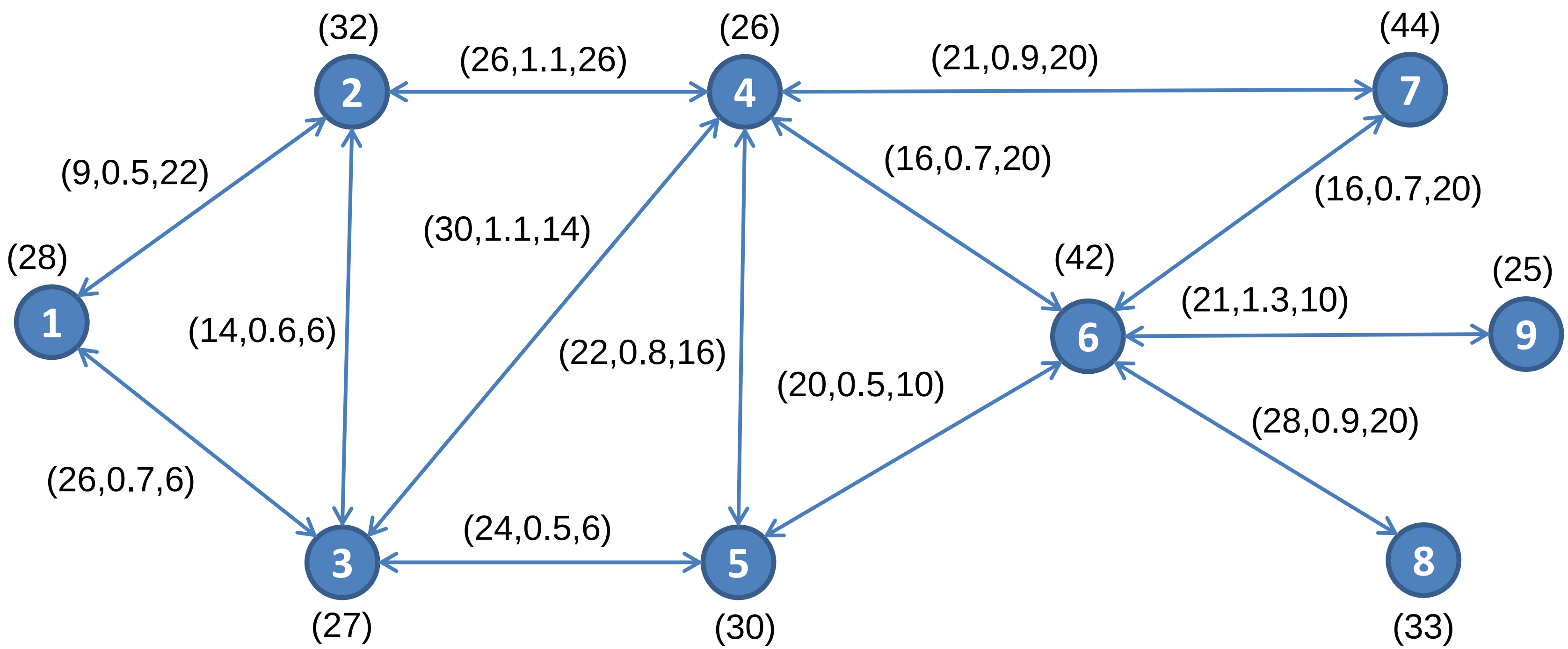
Using state of the art models: Z(pax) = (25 + **3**) x 10.000 + (5 + 1) x 100 = **280.600**



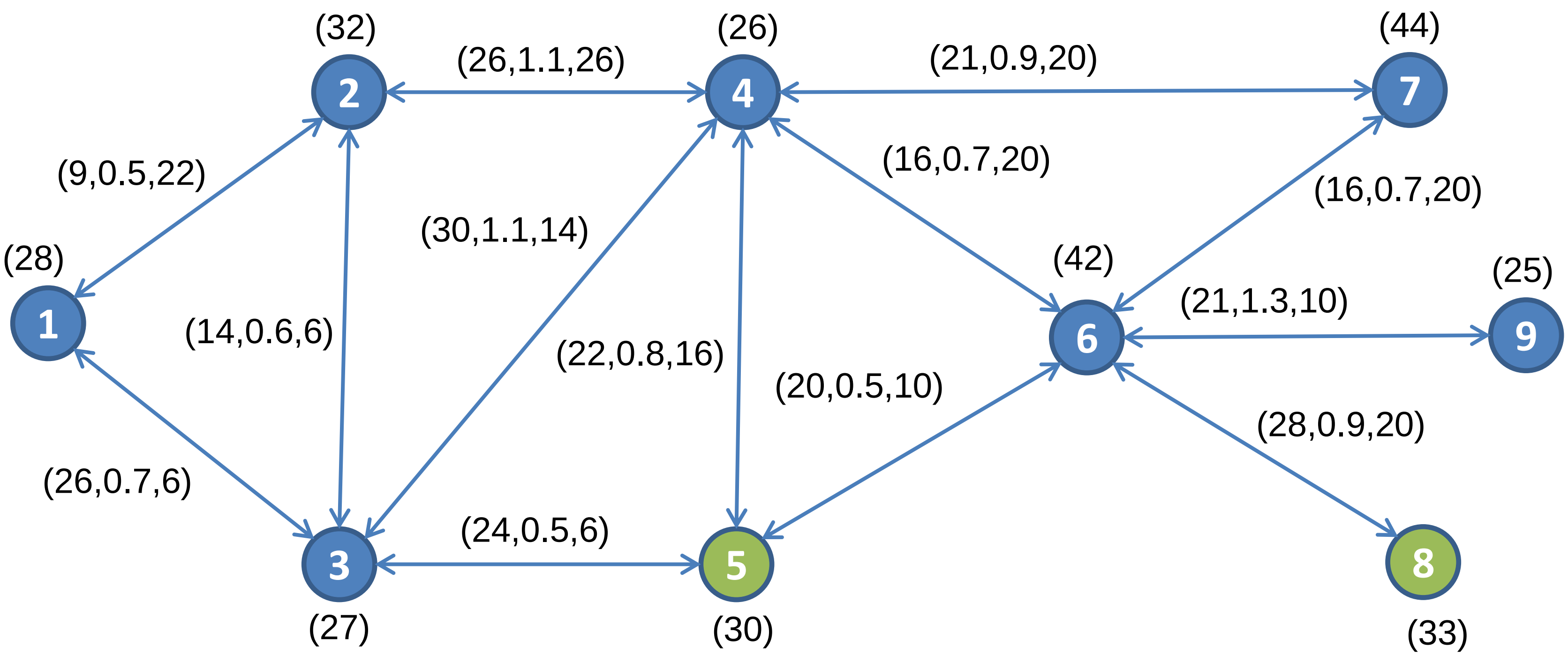
Appendix III

The procedure of enlarging the N2 Network

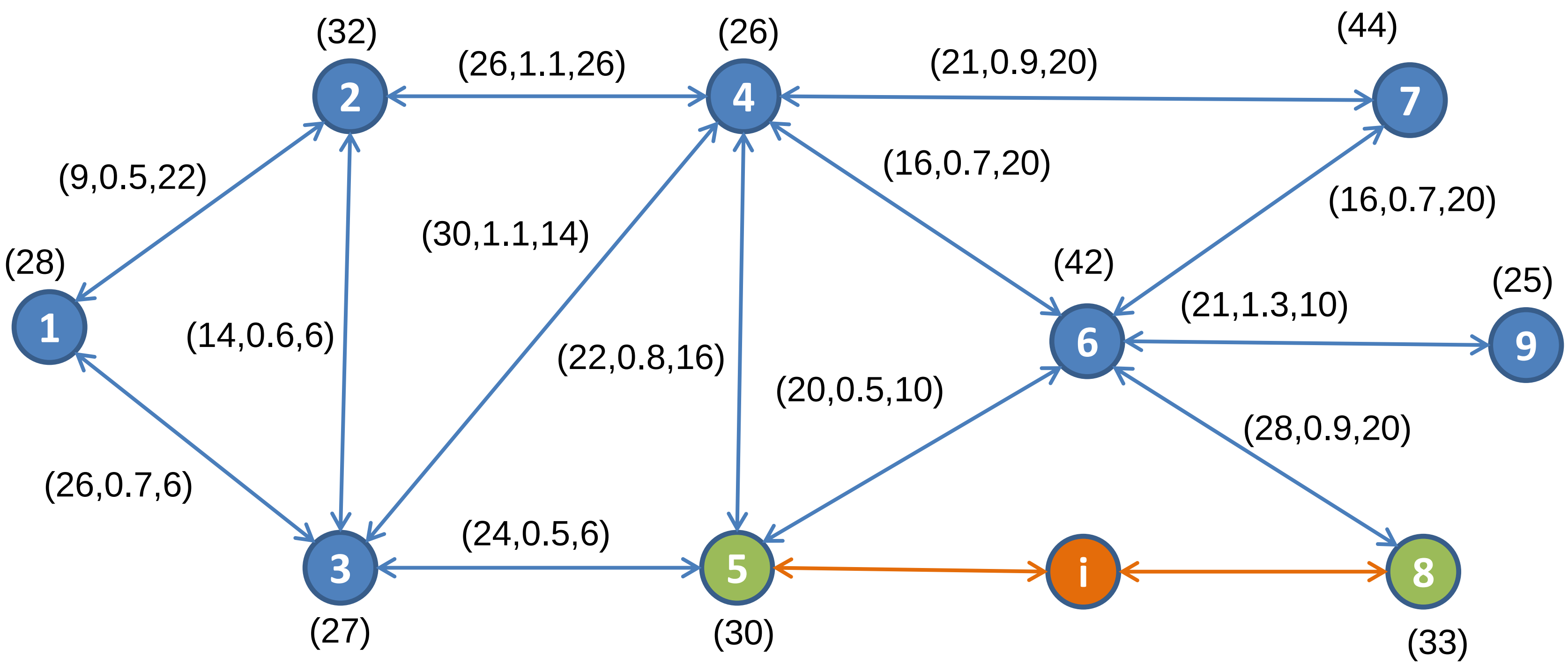
N2 is used as initial network for the Network Size vs. Performance Test



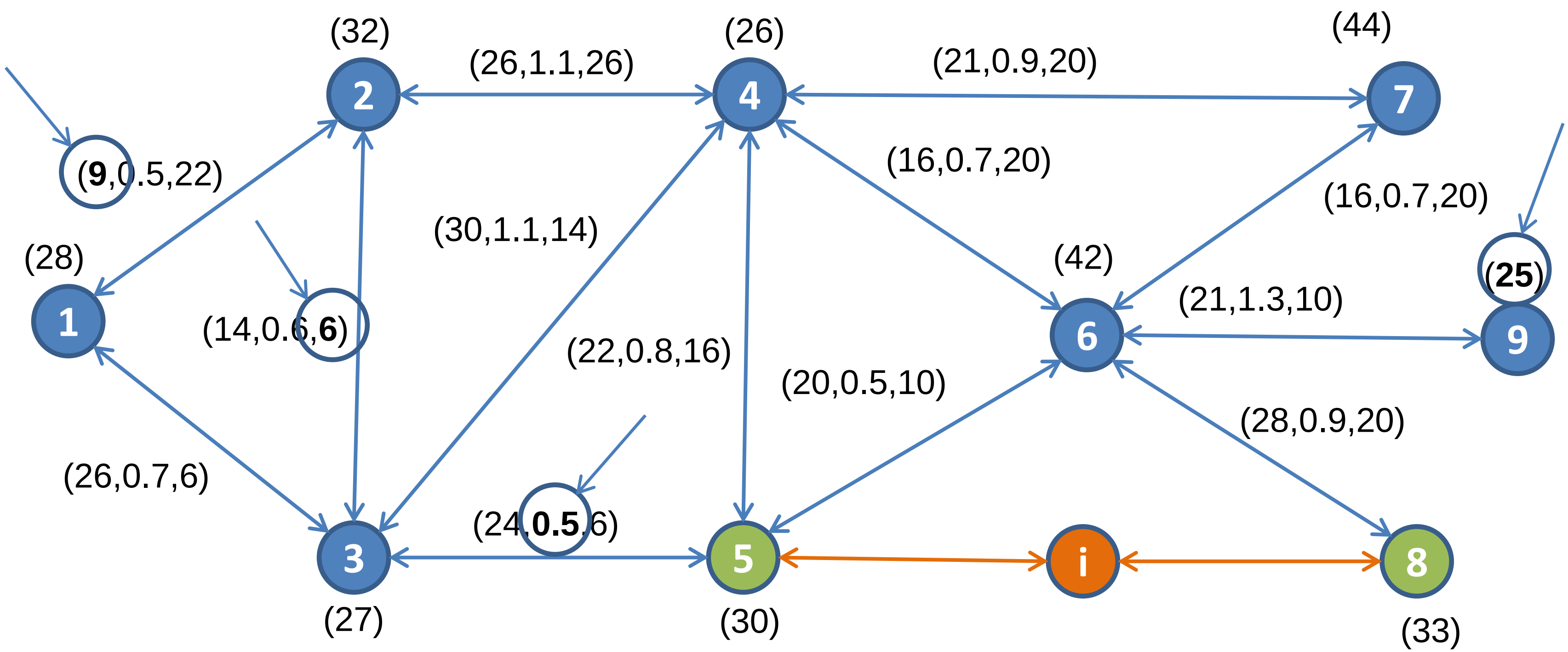
The algorithm selects two pair of nodes randomly....



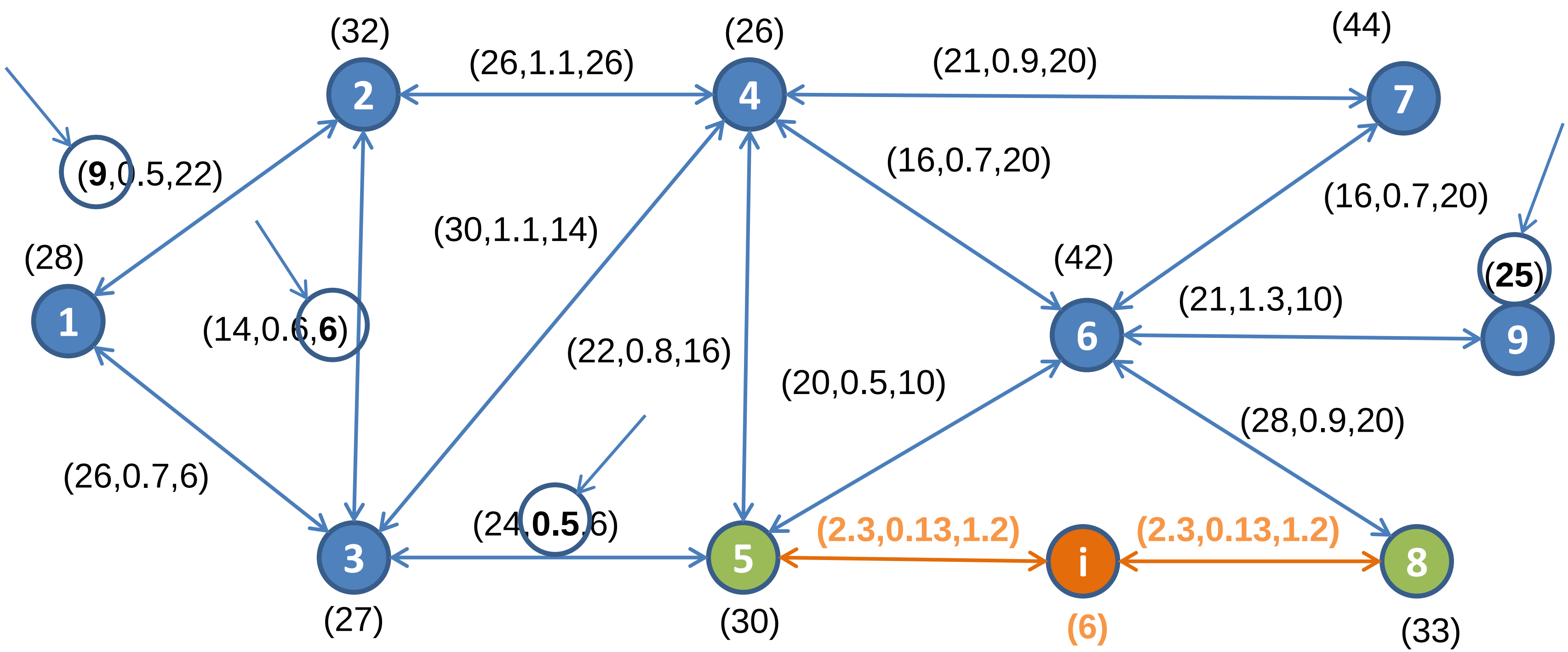
Then, a new node and two new stretches are created to link them



Finally, it looks for the minimum costs regarding each stretch and node component



which are selected and put on the new node and stretches as the ¼ part of their value



New Node features

$$\left(\frac{c^y_{il\min}}{4}\right) \quad \text{i} \quad c^y_i = 10 \cdot c^y_{il}$$

New stretch features

$$\left(\frac{c^x_{ijl\min}}{4}, \frac{t^{TP}_{ij\min}}{4}, \frac{c^z_{ij\min}}{4}\right) \quad c^x_{ij} = 10 \cdot \left(\frac{c^x_{ijl\min}}{4}\right)$$