A Fixed Point Route Choice Model for Route Correlation

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Contents

Route choice modelling

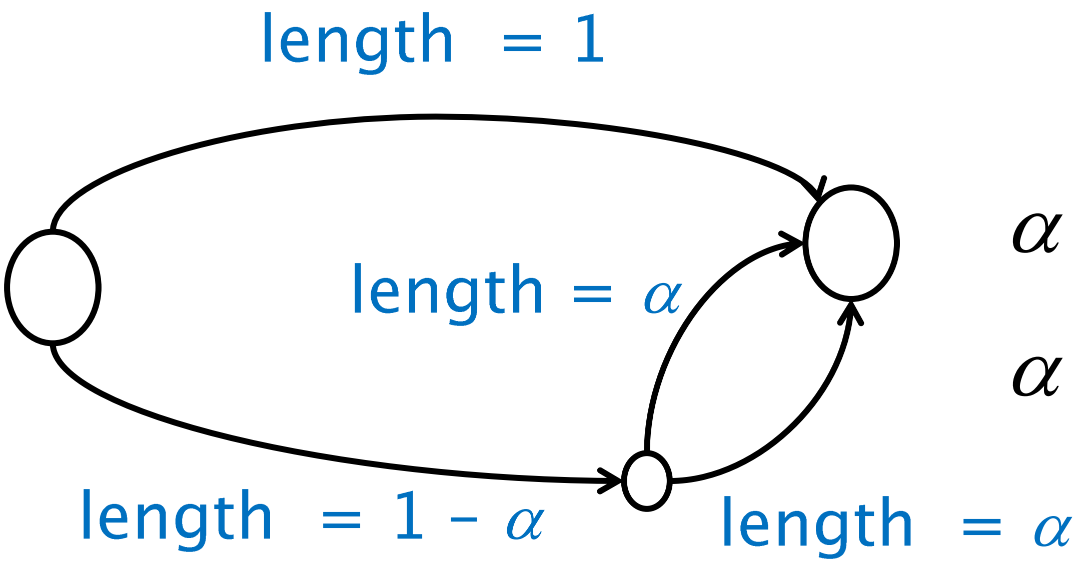
Traffic assignment and route choice models area a key stage of urban transportation planning

These models can be divided in 3 groups:

- Deterministic equilibrium models
- Stochastic/probabilistic assignment models
- Dynamic assignment models

Route correlation

Route correlation due to path overlapping is an important issue when modelling people's behaviour



$$\alpha \approx 1 \quad \Pr(up) \approx 1/3$$

$$\alpha \approx 0 \quad \Pr(up) \approx 1/2$$

This is a behavioural issue!

Dealing with path overlapping

C–Logit

$$\Pr_w^p = \frac{\exp\left(-\theta c_w^p + CF_w^p\right)}{\sum_{r \in p^w} \exp\left(-\theta c_w^r + CF_w^r\right)}$$

Path Size Logit

$$\Pr_w^p = \frac{\exp\left(-\theta c_w^p + \beta \cdot \ln PS_w^p\right)}{\sum_{r \in p^w} \exp\left(-\theta c_w^r + \beta \cdot \ln PS_w^r\right)}$$

Dealing with path overlapping

Proposed Fixed Point Logit

$$\Pr_w^p = \frac{h_w^p}{T_w}$$

$$\Pr_w^p = \frac{\exp\left(-\theta c_w^p - \rho \sum_{\substack{q \in p^w \\ q \neq p}} \left[\eta_w^{pq} h_w^q\right]\right)}{\sum_{r \in p^w} \exp\left(-\theta c_w^r - \rho \sum_{\substack{q \in p^w \\ q \neq p}} \left[\eta_w^{rq} h_w^q\right]\right)}$$

This model cannot be estimated using traditional methods

Model estimation

$$\Pr_w^{p\ t=0} = \frac{\exp\left(-\theta^{t=0} c_w^p\right)}{\sum_{r \in p^w} \exp\left(-\theta^{t=0} c_w^r\right)} \longrightarrow h_w^{p\ t=0}$$

$$\Pr_w^{p\ t=1} = \frac{\exp\left(-\theta^{t=1} c_w^p - \rho^{t=1} \sum_{\substack{q \in p^w \\ q \neq p}} \left[\eta_w^{pq} h_w^{q\ t=0}\right]\right)}{\sum_{r \in p^w} \exp\left(-\theta^{t=1} c_w^r - \rho^{t=1} \sum_{\substack{q \in p^w \\ q \neq p}} \left[\eta_w^{rq} h_w^{q\ t=0}\right]\right)} \longrightarrow h_w^{p\ t=1}$$

Model estimation

For any iteration n:

$$\Pr_w^{p\ t=n} = \frac{\exp\left(-\theta^{t=n} c_w^p - \rho^{t=n} \sum_{\substack{q \in p^w \\ q \neq p}} \left[\eta_w^{pq} h_w^{q\ t=n-1} \right]\right)}{\sum_{r \in p^w} \exp\left(-\theta^{t=n} c_w^r - \rho^{t=n} \sum_{\substack{q \in p^w \\ q \neq p}} \left[\eta_w^{rq} h_w^{q\ t=n-1} \right]\right)}$$

Repeat until:

$$\theta^{t=n} \approx \theta^{t=n-1}$$

$$\rho^{t=n} \approx \rho^{t=n-1}$$

Marginal rates of substitution

$$c_w^p = \sum_k \theta_k X_{w,k}^p$$

For the proposed Fixed Point Logit

$$\frac{\partial V_w^p}{\partial X_{w,k}^p} = \frac{\theta_k}{1 - \dfrac{1}{T_w} \cdot \rho \sum\limits_{\substack{q \in p^w \\ q \neq p}} \eta_w^{pq} \cdot h_w^p \cdot h_w^q}$$

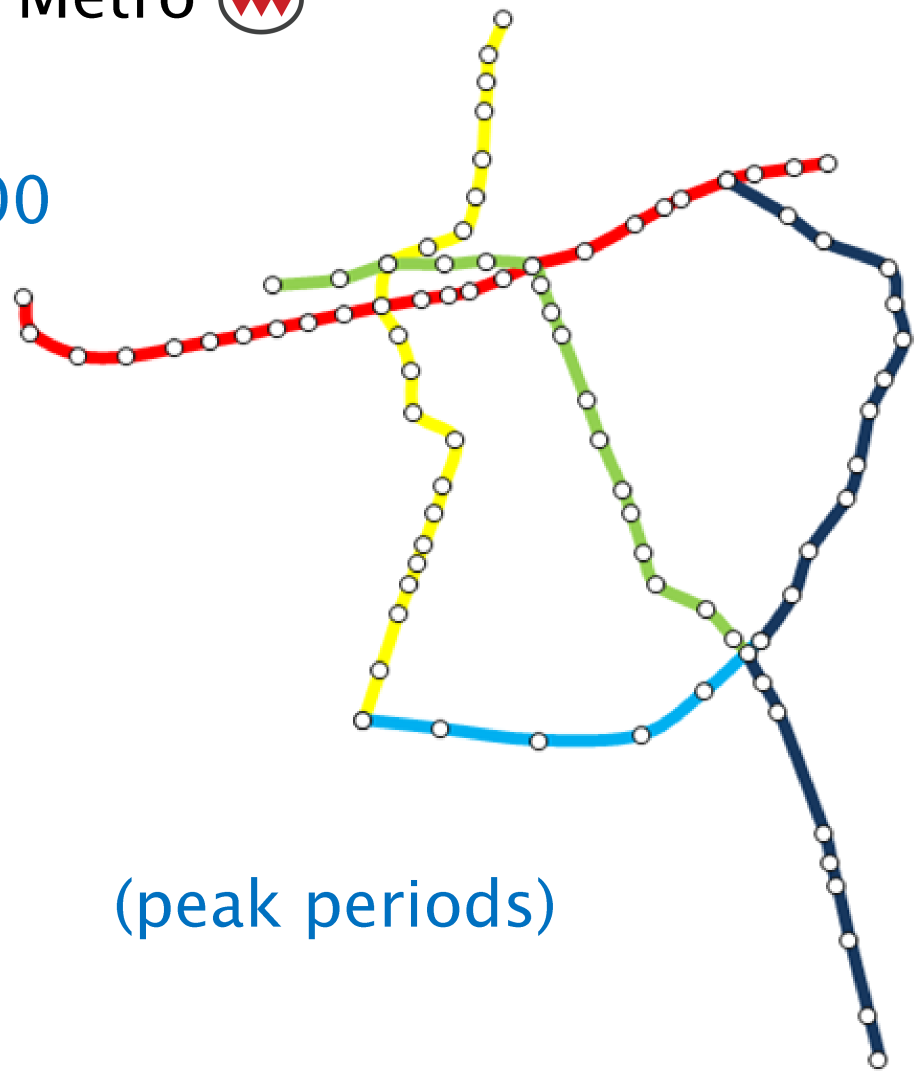
The marginal rates of substitution have the form

$$\frac{\partial V_r \big/ \partial X_{w,k_1}^p}{\partial V_r \big/ \partial X_{w,k_2}^p} = \frac{\theta_{k_1}}{\theta_{k_2}}$$

Study Case

Route choice on the Santiago Metro

Santiago Metro



Daily trips	2,300,000	
Survey date	2008	
Length	78 Km	
Lines	5	
Stations	85	
Survey size	16,029	(peak periods)

Set of alternative routes

A key element when dealing with probabilistic route choice models is the definition of the alternatives for the OD pairs of interest

generated based on the actual choices
→ 2 to 4 alternative routes

The systematic utility function consisted of total trip time and number of transfers

Models' specification

We estimated a Multinomial Logit (MNL), a C-Logit (CL), a Path Size Logit (PSL) and the proposed Fixed Point Logit (FPL)

The spatial correlation factor due to overlapping for the FPLM is (Yai et al, 1997):

$$\eta_w^{pq} = \frac{D_w^{pq}}{\sqrt{D_w^p \cdot D_w^q}}$$

Estimation results

Attribute	MNL	CL	PSL	FPL
Trip Time	−0.141 (−47.20)	−0.142 (−44.68)	−0.141 (−43.96)	−0.117 (−37.15)
Transfers	−1.140 (−48.44)	−1.144 (−48.43)	−1.144 (−48.42)	−0.924 (−35.36)
Spatial Correlation	n.a.	−0.081 (−0.57) *	−0.101 * (−0.68) *	−0.062 (−14.98)
Log-Likelihood	−7.381	−7.380	−7.380	−7.235
Adjusted ρ^2	0.362	0.362	0.362	0.374

The proposed FPL converged to a 0.01% tolerance (1 over 10,000) in only 4 iterations

Results and analysis

The FPL model is the only one able to capture correlation between routes

The FPL model presents a significant improvement on goodness-of-fit and forecasting capability

Marginal rate of substitution	MNL	8.1 min/transfer
	FPL	7.9 min/transfer

Summing up…

The results from the proposed model are satisfactory and superior to traditional route choice models

Although the formulation is more complex, obtaining marginal rates of substitution is simple and the iterative estimation process converges rapidly

The proposed model can be extended to congested networks (i.e. private transportation) and should be tested in larger networks

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