Ferry Service Network Design Under Demand Uncertainty

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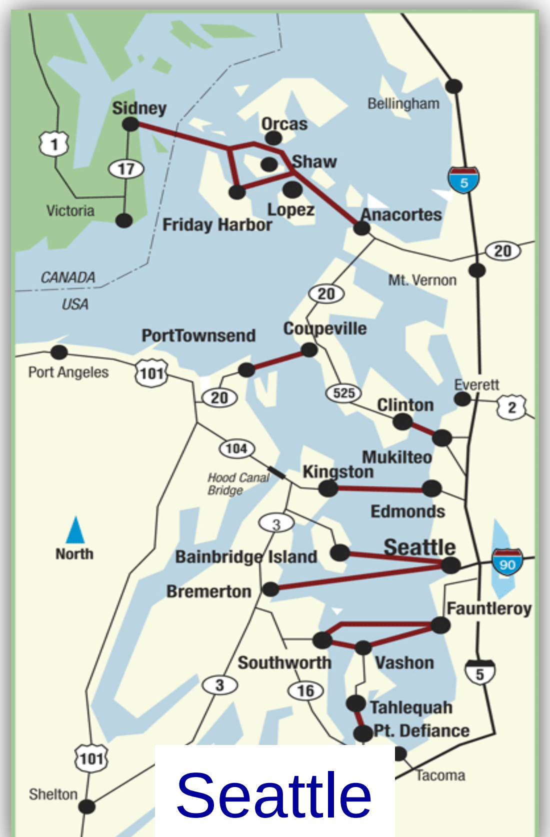
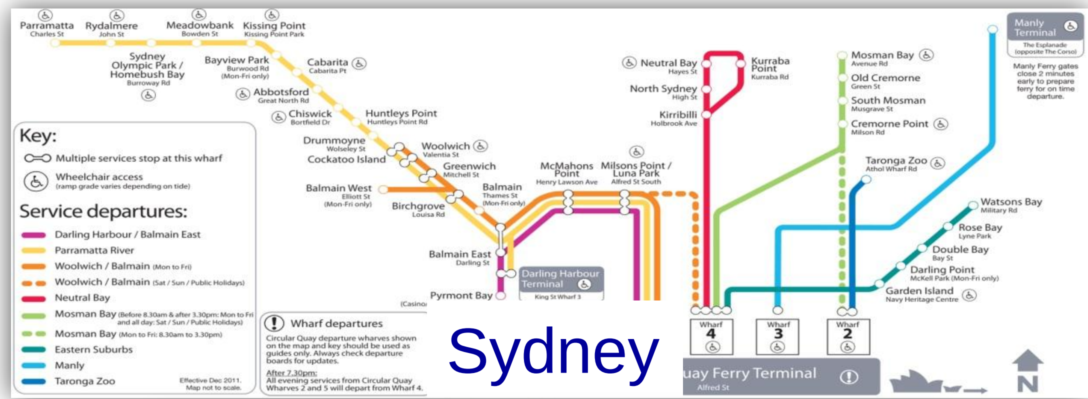
Background

- Ferries form a part of public transport systems of many water side cities and islands.





Background

- Hong Kong ferry service(journey time/ headway)
  - Cross-harbor (10 min/ 10-20 min)
  - Outlying islands (60 min/ 30-120 min)
  - Macau and Mainland China (>60min/--)
- Characters
  - Some routes are profitable whereas others to remote islands are not due to low load factor
- Network design
  - Maximize profits while providing acceptable services

Background

- Service network design problem(SNDP)
  - Determine service network--------simultaneously--------passenger flow ➡ objective
  - First formulated by Magnanti (1984) as a mixed integer linear problem (MILP)
- Application
  - Ferry-fleet management by Lai and Lo (2004)
  - Wang and Lo (2009) formulated the problem with different service types
- Stochastic SNDP
  - Stochastic demand
  - Typically formed as a two-stage problem solved by L-shaped/Multi-cut method

Content

- Model formulation
  - Deterministic Formulation (P0)
  - Two-stage Stochastic Formulation (P1)
  - Service Reliability based Stochastic Formulation (P2)
- Solution procedure
- Numerical studies
- Conclusion

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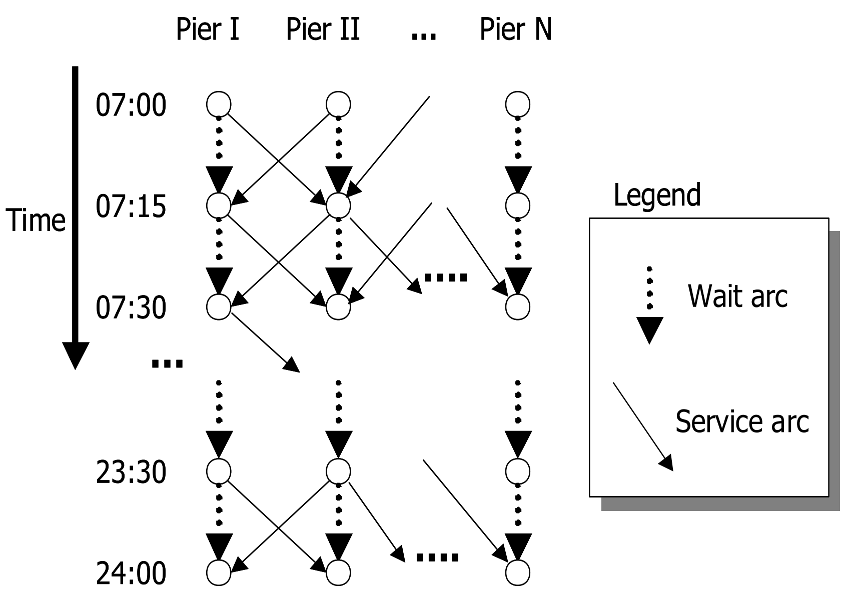
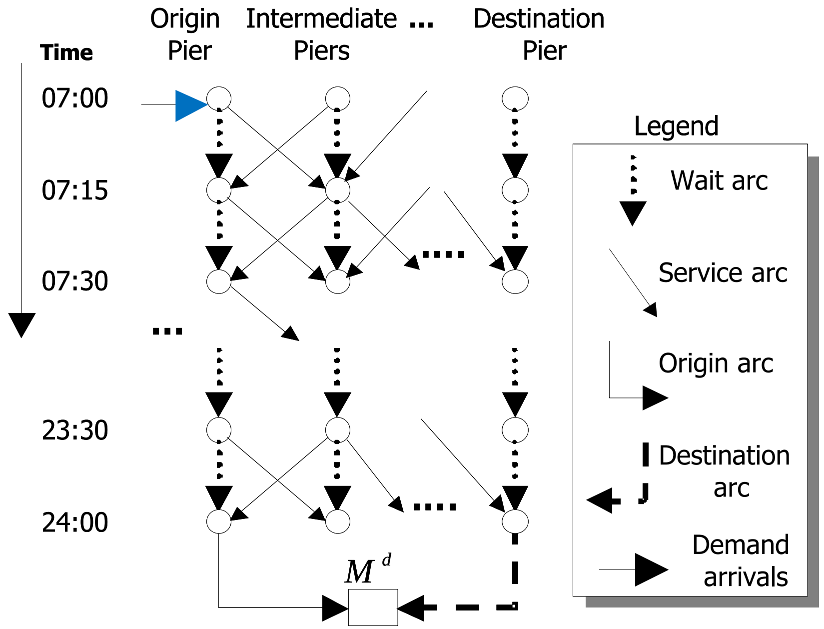
Network description

- Ferry time-space network



- Passenger time-space network



- Notations

$\mathbf{Y} = \{Y_{ij}\}, ij \in \begin{Bmatrix} S^f \\ W^f \end{Bmatrix}$ Ferry service arc / Ferry waiting arc

$\mathbf{X} = \{X_{ij}^d\}, ij \in \begin{Bmatrix} S^d \\ W^d \end{Bmatrix}$ Passenger service arc / Passenger waiting arc

Deterministic Formulation(P0)

$$\min_{\mathbf{Y},\mathbf{X}} \overbrace{\boxed{\sum_{i\in N_b^f}\ \sum_{j\in N^f\setminus N_b^f} Y_{ij}F}}^{\text{Fixed cost}} + \overbrace{\boxed{\sum_{ij\in S^f} Y_{ij}C_{ij}}}^{\text{operating cost}} + \overbrace{\boxed{\sum_{d\in R}\ \sum_{ij\in W^d} X_{ij}^d P_{ij}^d}}^{\text{Passenger waiting cost}}$$

➢Ferry flow conservation:

$$\sum_{j\in N^f} Y_{ij} - \sum_{k\in N^f} Y_{ki} = 0 \quad \forall i\in N^f \setminus \left(N_b^f \cup N_e^f\right)$$

➢Fleet size:

$$\sum_{i\in N_b^f}\ \sum_{j\in N^f\setminus N_b^f} Y_{ij} \le V$$

➢Upper bound of ferry flow:

$$0 \le Y_{ij} \le U_{ij}^f \quad \forall ij\in A^f$$

$$Y_{ij} \in \text{integer} \quad \forall ij\in A^f$$

➢Relationship between ferry and passenger flows:

$$\sum_{d\in R} X_{ij}^d \le Y_{ij}\zeta \quad \forall ij\in S^f$$

➢Passenger conservation:

$$\sum_{j\in N^d} X_{ij}^d - \sum_{k\in N^d} X_{ki}^d = \begin{cases} B^d & \text{if } i \text{ is the origin} \\ 0 & \text{otherwise} \end{cases}$$

Stochastic demand

- Assumption
  - Follow some known distribution
- Two service type
  - Regular services vs. ad-hoc services
  - Fixed schedule vs. flexible schedule

Two-stage Stochastic Formulation(P1)

- Ad-hoc service

$\boxed{\mathbf{Z} = \left\{ Z^d \right\}}$

- Regular service

$\boxed{\mathbf{Y} = \left\{ Y_{ij} \right\}}$

- Passenger flow

$\boxed{\mathbf{X} = \left\{ X_{ij}^d \right\}}$

$$\min_{\mathbf{X},\mathbf{Y},\mathbf{Z}} \Phi(\mathbf{Y}) = \sum_{i \in N_b^f} \sum_{j \in N^f \setminus N_b^f} Y_{ij} F + \sum_{ij \in S^f} Y_{ij} C_{ij} + \overline{Q}(\mathbf{Y})$$

$$s.t. \sum_{j \in N^f} Y_{ij} - \sum_{k \in N^f} Y_{ki} = 0 \quad \forall i \in N^f \setminus \left(N_b^f \cup N_e^f \right)$$

$$\sum_{i \in N_b^f} \sum_{j \in N^f \setminus N_b^f} Y_{ij} \le V$$

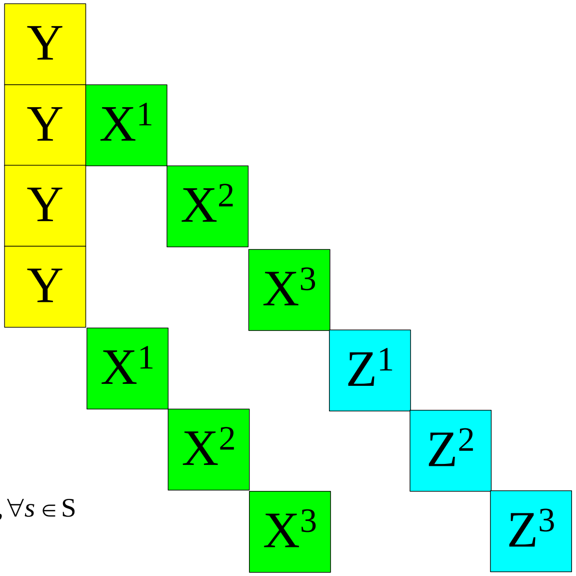
$$0 \le Y_{ij} \le U_{ij}^f \quad \forall ij \in A^f$$

$$Y_{ij} \in \text{integer} \quad \forall ij \in A^f$$

where $$\overline{Q}(\mathbf{Y}) = \min \sum_{s \in S} p_s \left(AC \sum_{d \in R} Z_s^d + \sum_{d \in R} \sum_{ij \in W^d} X_{ij,s}^d P_{ij}^d \right)$$

$$\sum_{d \in R} X_{ij,s}^d \le Y_{ij} \zeta \qquad \forall ij \in S^f, \quad \forall s \in S$$

$$\sum_{j \in N^d} X_{ij,s}^d - \sum_{k \in N^d} X_{ki,s}^d = \boxed{\begin{cases} B_s^d - Z_s^d \\ 0 \end{cases}} \begin{matrix} \text{if } i \text{ is the origin of } d \\ \text{otherwise} \end{matrix} \quad \forall d \in \mathrm{R}, \forall s \in \mathrm{S}$$



Solution procedure for P1

- L-shaped/ multi-cut method
  - Master problem produces lower bound
  - Sub problem produces upper bound as well as generates cuts for master problem
- Characters
  - The master problem grows as the procedure proceeds
  - The global optimal solution is not guaranteed

Reliability-based Stochastic Formulation(P2)

- Service reliability r
  - Two types: regular services and ad-hoc services
  - Service reliability: the probability of passengers carried by regular services

Service reliability	regular services	ad-hoc services
high	more	less
low	less	more

Reliability-based Stochastic Formulation(P2)

$$\min_{X,Y,Z,\rho} \phi(\boldsymbol{\rho}) = \sum_{i \in N_b^f} \sum_{j \in N^f \setminus N_b^f} Y_{ij} F + \sum_{ij \in S^f} Y_{ij} C_{ij} + \bar{Q}$$

Phase-1

$$\min_{X,Y} g = \sum_{i \in N_b^f} \sum_{j \in N^f \setminus N_b^f} Y_{ij} F + \sum_{ij \in S^f} Y_{ij} C_{ij} + \sum_{d \in R} \sum_{ij \in W^d} X_{ij}^d P_{ij}^d$$

$$s.t. \quad \sum_{j \in N^f} Y_{ij} - \sum_{k \in N^f} Y_{ki} = 0 \qquad \forall i \in N^f \setminus (N_b^f \cup N_e^f)$$

$$\sum_{i \in N_b^f} \sum_{j \in N^f \setminus N_b^f} Y_{ij} \le V$$

$$0 \le Y_{ij} \le U_{ij}^f$$

$$\boxed{\begin{aligned} &\sum_{j \in N^d} X_{ij}^d - \sum_{k \in N^d} X_{ki}^d = \begin{cases} \bar{B}^d & \text{if } i \text{ is the origin} \\ 0 & \text{otherwise} \end{cases} \quad \forall d \in R, i \in N^d \\ &\bar{B}^d = \Psi_d^{-1}\left(\rho^d\right) \end{aligned}}$$

$$\sum_{d \in R} X_{ij}^d \le Y_{ij} \zeta \qquad \forall ij \in S^f$$

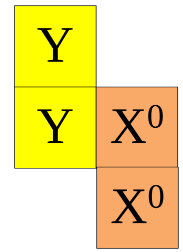
$$\bar{Q} = \sum_{s \in S} p_s Q_s$$

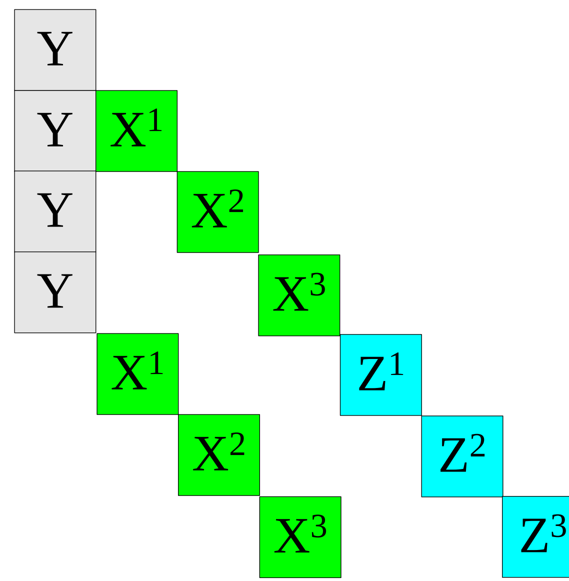
Phase-2

$$\min_{X,Z} Q_s = AC \sum_{d \in R} Z_s^d + \sum_{d \in R} \sum_{ij \in W^d} X_{ij,s}^d P_{ij}^d \qquad \forall s \in \mathrm{S}$$

$$s.t. \quad \sum_{j \in N^d} X_{ij,s}^d - \sum_{k \in N^d} X_{ki,s}^d = \begin{cases} B_s^d - Z_s^d & \text{if } i \text{ is the origin} \\ 0 & \text{otherwise} \end{cases} \quad \forall d \in \mathrm{R}$$

$$\sum_{d \in R} X_{ij,s}^d \le Y_{ij} \zeta \qquad \forall ij \in S^f$$





Reliability-based Stochastic Formulation(P2)

$$\min_{\mathbf{X},\mathbf{Y},\mathbf{Z},\boldsymbol{\rho}} \phi(\boldsymbol{\rho}) = \sum_{i\in N_b^f}\sum_{j\in N^f\setminus N_b^f} Y_{ij}F + \sum_{ij\in S^f} Y_{ij}C_{ij} + \bar{Q}(\boldsymbol{\rho})$$

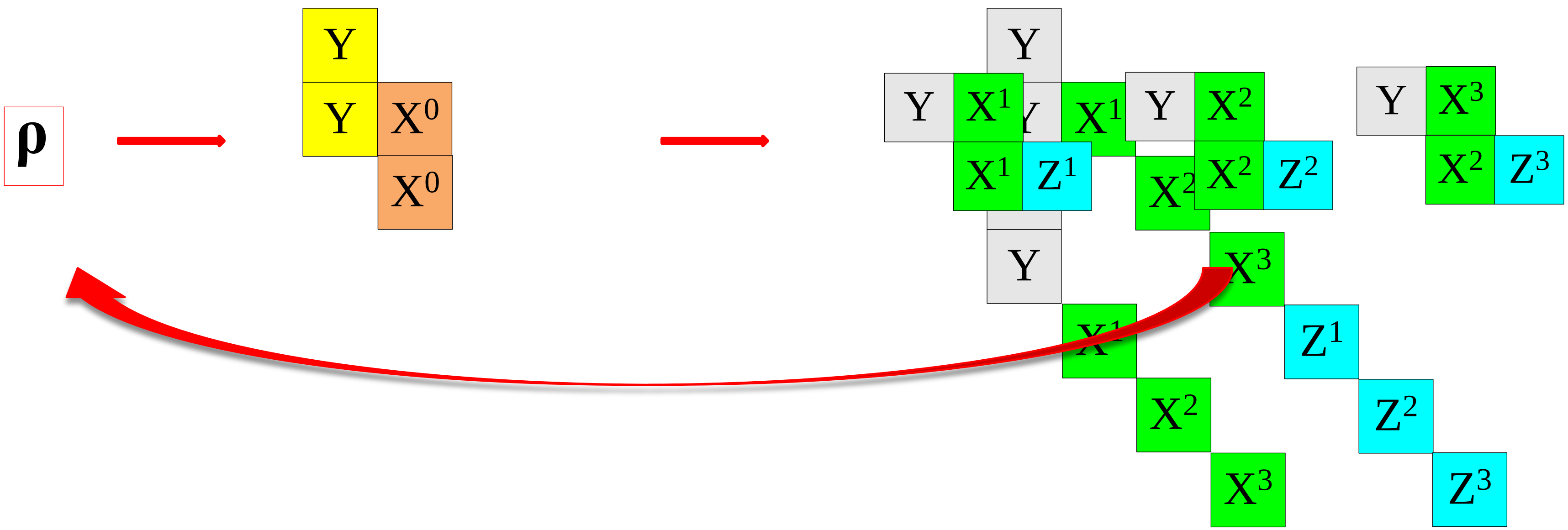
Phase-1

$$\min_{\mathbf{X},\mathbf{Y}} \; g = \sum_{i\in N_b^f}\sum_{j\in N^f\setminus N_b^f} Y_{ij}F + \sum_{ij\in S^f} Y_{ij}C_{ij} + \sum_{d\in R}\sum_{ij\in W^d} X_{ij}^{d}P_{ij}^{d}$$

Phase-2

$$\bar{Q} = \sum_{s\in S} p_s Q_s$$

$$\min_{\mathbf{X},\mathbf{Z}} \; Q_s = AC\sum_{d\in R} Z_s^d + \sum_{d\in R}\sum_{ij\in W^d} X_{ij,s}^{d}P_{ij}^{d} \qquad \forall s\in S$$



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  - Deterministic Formulation (P0)
  - Two-stage Stochastic Formulation (P1)
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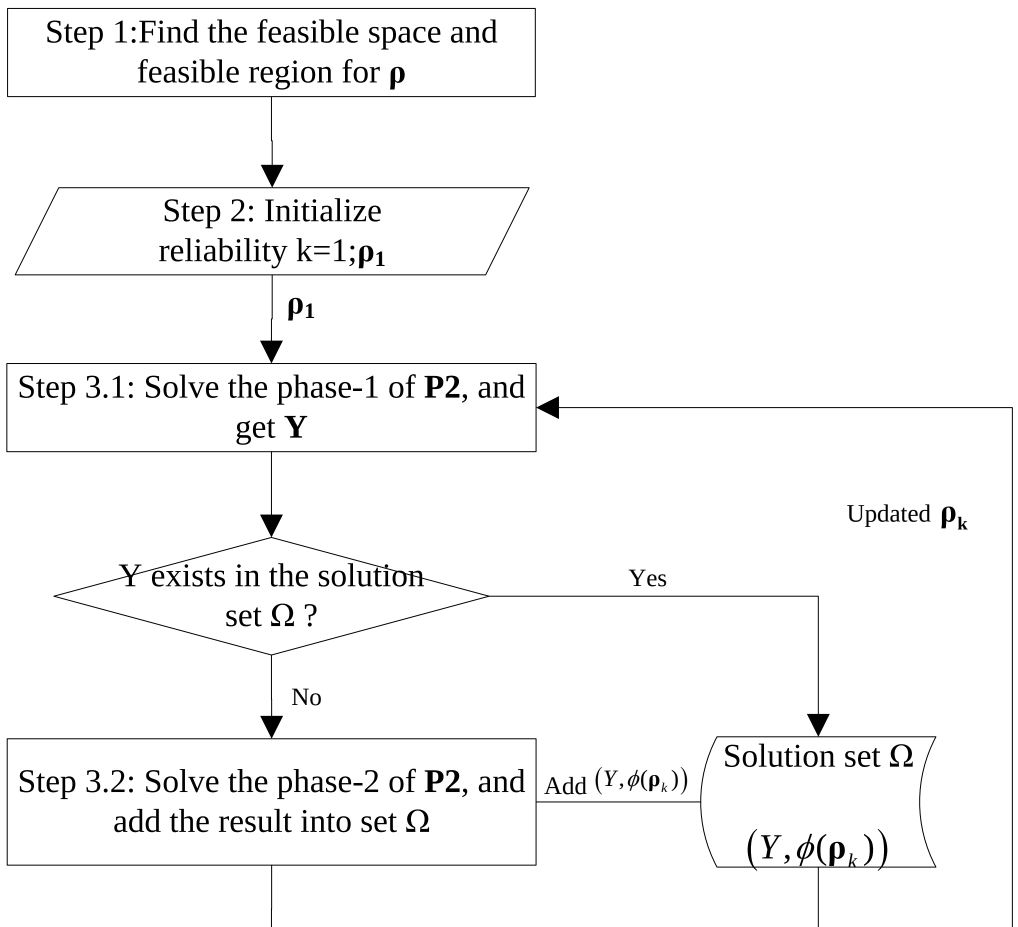
Solution procedure

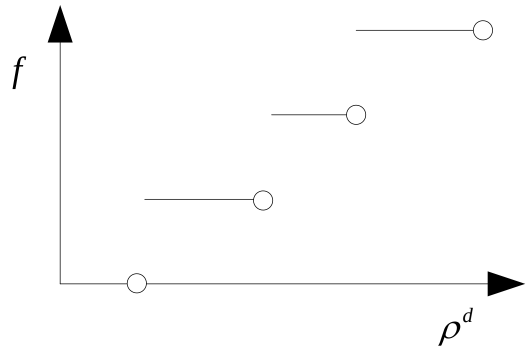
- SR-based gradient approach

The set of solutions is denoted as Ω



Step 5: get $\nabla\phi(\boldsymbol{\rho}_k)$

- *Discrete character of* Y_{ij}



Step 5: get $\nabla\phi(\boldsymbol{\rho}_k)$

- Method:
  1) increase ρ^1 by a small number each time until $\mathbf{Y}$ changes.
  2) When ρ^1 cannot be increased any more, decrease it.
  - *Use the sensitivity of* ρ^1

$$\frac{\Delta\phi(\boldsymbol{\rho}_k)}{\Delta\rho^1} = \frac{\phi(\boldsymbol{\rho}_k)^{(2)} - \phi(\boldsymbol{\rho}_k)^{(1)}}{\rho^{1(2)} - \rho^{1(1)}}$$

  - *Finally, formulate*

$$\nabla\phi(\boldsymbol{\rho}_k)$$

For simplicity, we adopt the partial derivative symbols.

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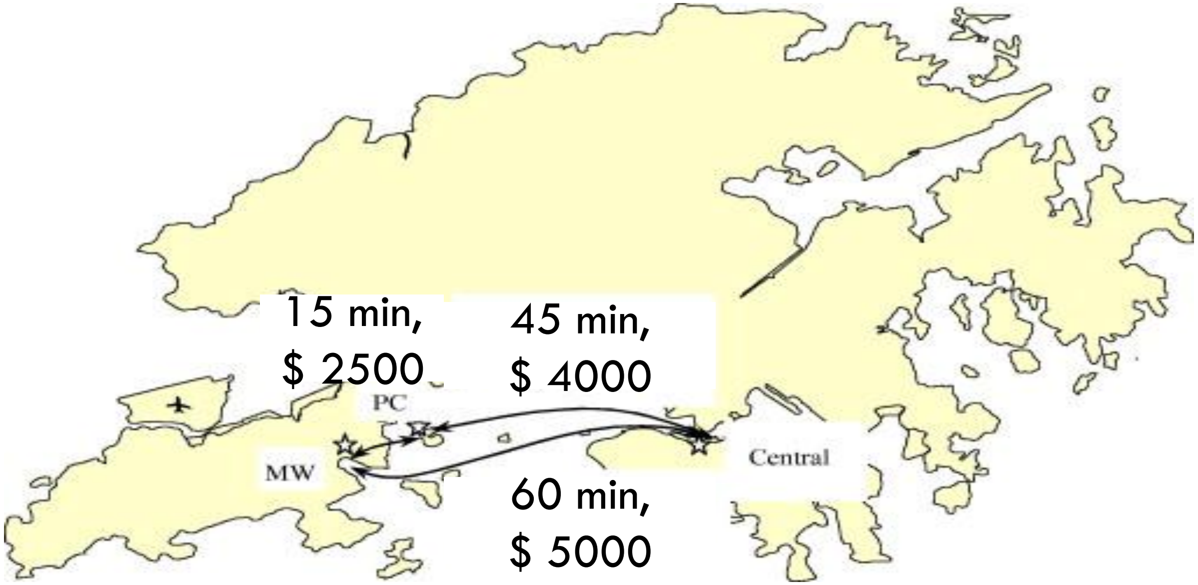
Numerical studies

- A ferry service network in Hong Kong



Time Slice	CBD-MW	MW-CBD	CBD-PC	PC-CBD	MW-PC	PC-MW
1	80	0	60	0	0	0
2	250	45	230	20	30	75
3	420	100	400	80	80	100
4	350	140	330	140	140	180
5	200	130	180	130	190	230
6	100	120	80	120	320	170
7	90	110	70	70	210	70
8	50	50	30	50	100	50

Sampling method and sample size

- Latin Hypercube sampling, Sample size: 100
- Chi-square test
  - Whether the samples are uniformly distributed

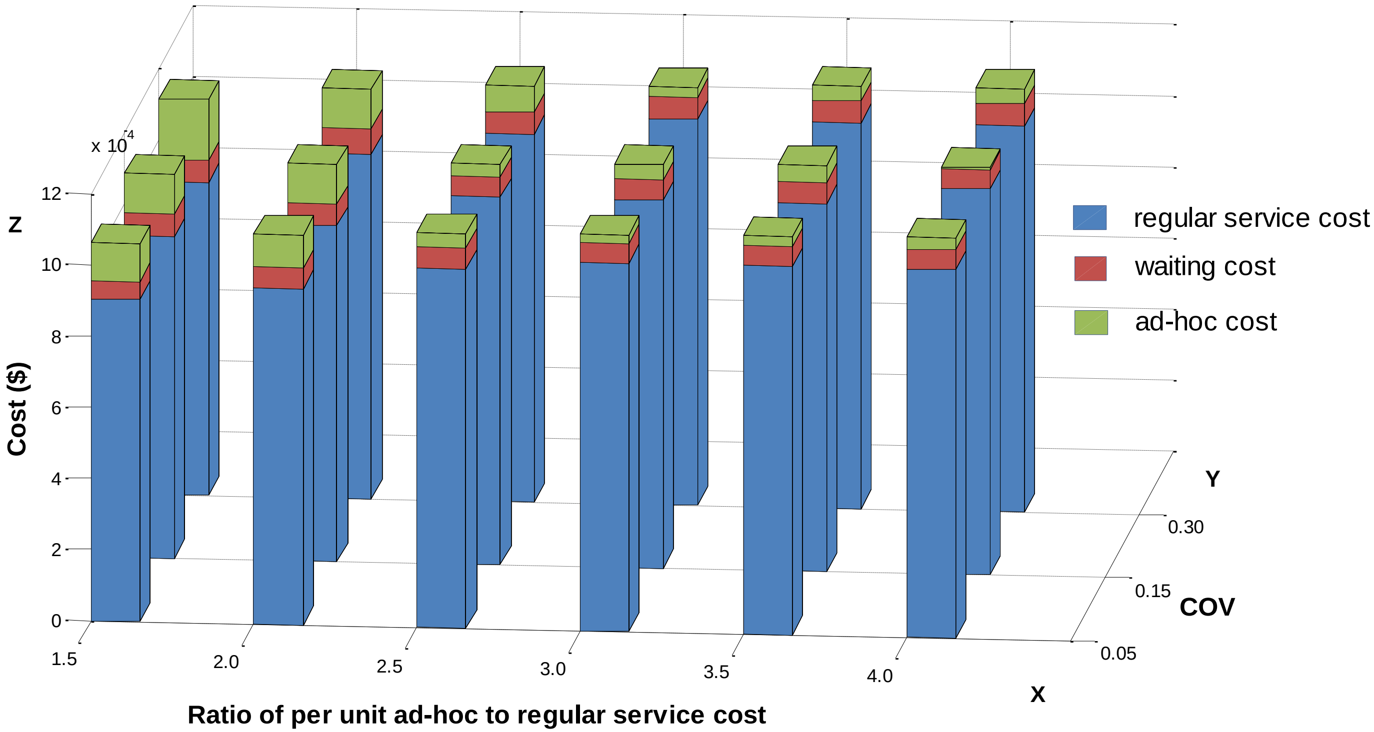
M	10	50	100	200	500
$E(\lvert\bar{X}-\mu\rvert)$	0.96	0.09	0.03	0.01	0.00
$E(\lvert S^2-\sigma^2\rvert)$	111.60	9.88	3.29	1.39	0.35
χ^2	20	4	0	0	0
P-value	0.30	0.999	1.000	1.000	1.000

- Variance test of the outcome of the examples
  - Test the outcomes of the transformation function, phase-2.

M	10	50	100	200	500
$E(h(\mathbf{X}))$	288011.9	287969.1	287970.6	287969.1	287970.1
$SD(h(\mathbf{X}))$	593.0	46.1	15.0	5.8	1.5
$[SD(h(\mathbf{X}))]/[E(h(\mathbf{X}))]$	2.06E-03	1.60E-04	5.21E-05	2.01E-05	5.21E-06
Time (s)	13.7	72.0	125.2	258.0	872.0

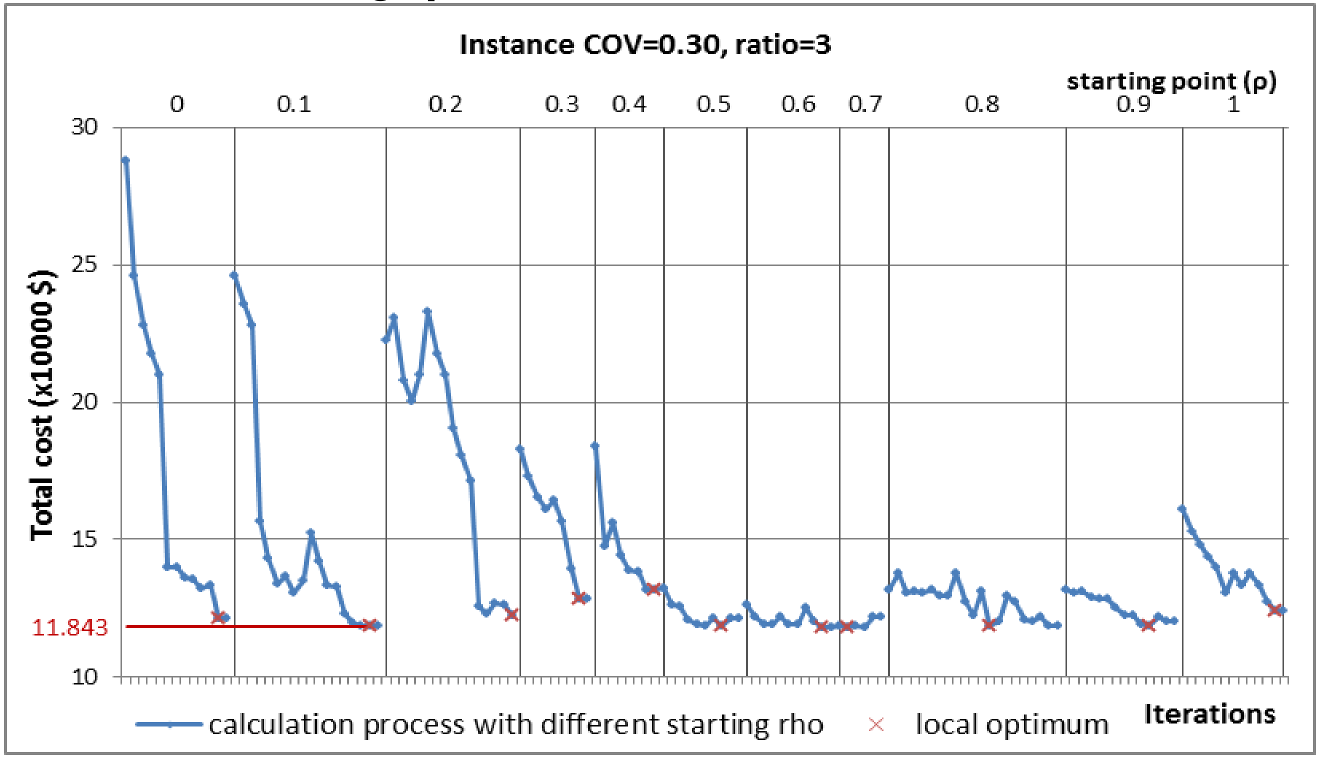
Result analysis

The composition of total cost versus ad-hoc cost ratio



Result analysis

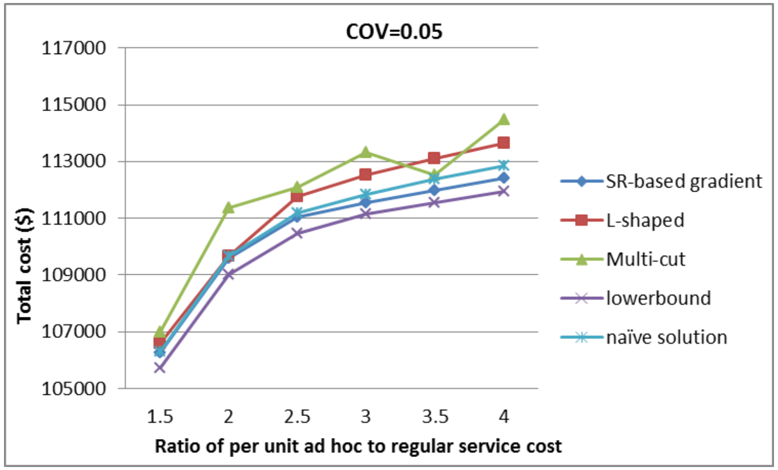
Objective function values versus iteration numbers for different starting points

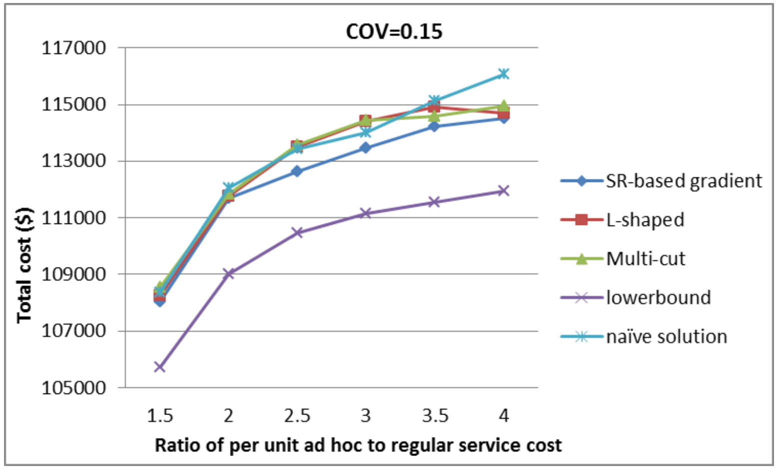


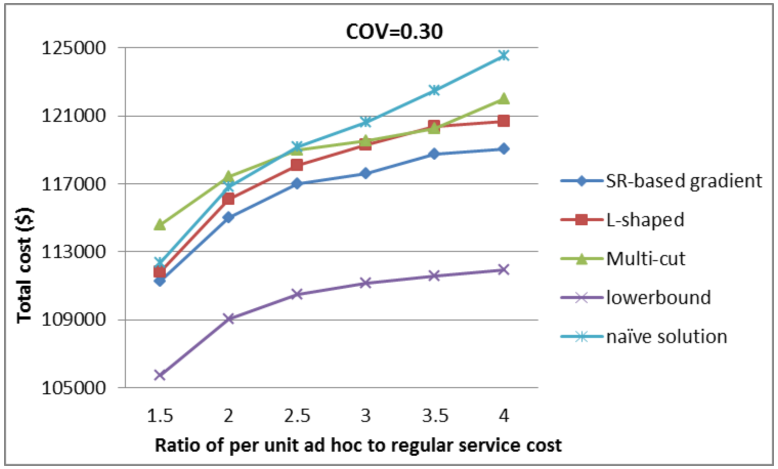
Result analysis

- SR-based gradient approach
- L-shaped method
- Multi-cut method
- Lower bound: replace the random demand by its expectation value (Jensen inequality)
- Native solution: use the Y generated from the lower bound and find the actual cost.

Result analysis







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Conclusion

- Formulate a reliability based service network design model under demand uncertainty
  - Use $\boldsymbol{\rho}$ as a tool to separate the two phases
  - Use sensitivity to get the decent direction
- Numerical studies
  - Realize the algorithm
  - Compare results with L-shaped and Multi-cut methods
- Future work
  - Passenger behavior in phase-2, UE assignment
  - Extend it to the rapid bus design

Thanks for your attention

Q & A